Recent Advances in Experiments and Modeling of Grid-forming Systems

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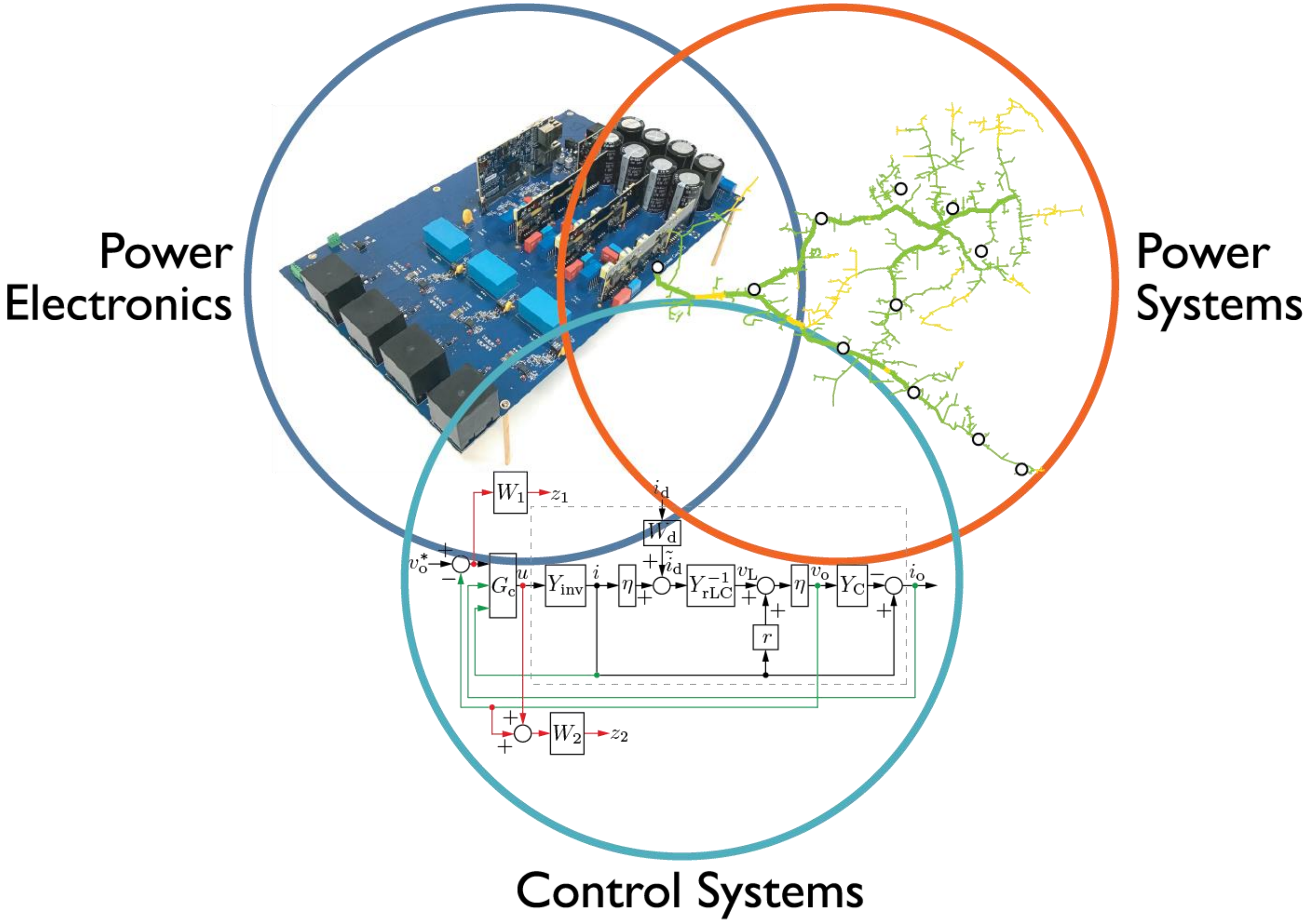
My Lab Activities



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Pranav

Debjyoti

and generous support from DOE & NSF for my research on:

- Grid-forming systems
- Power electronics
- UNIFI Consortium

SOLAR ENERGY TECHNOLOGIES OFFICE
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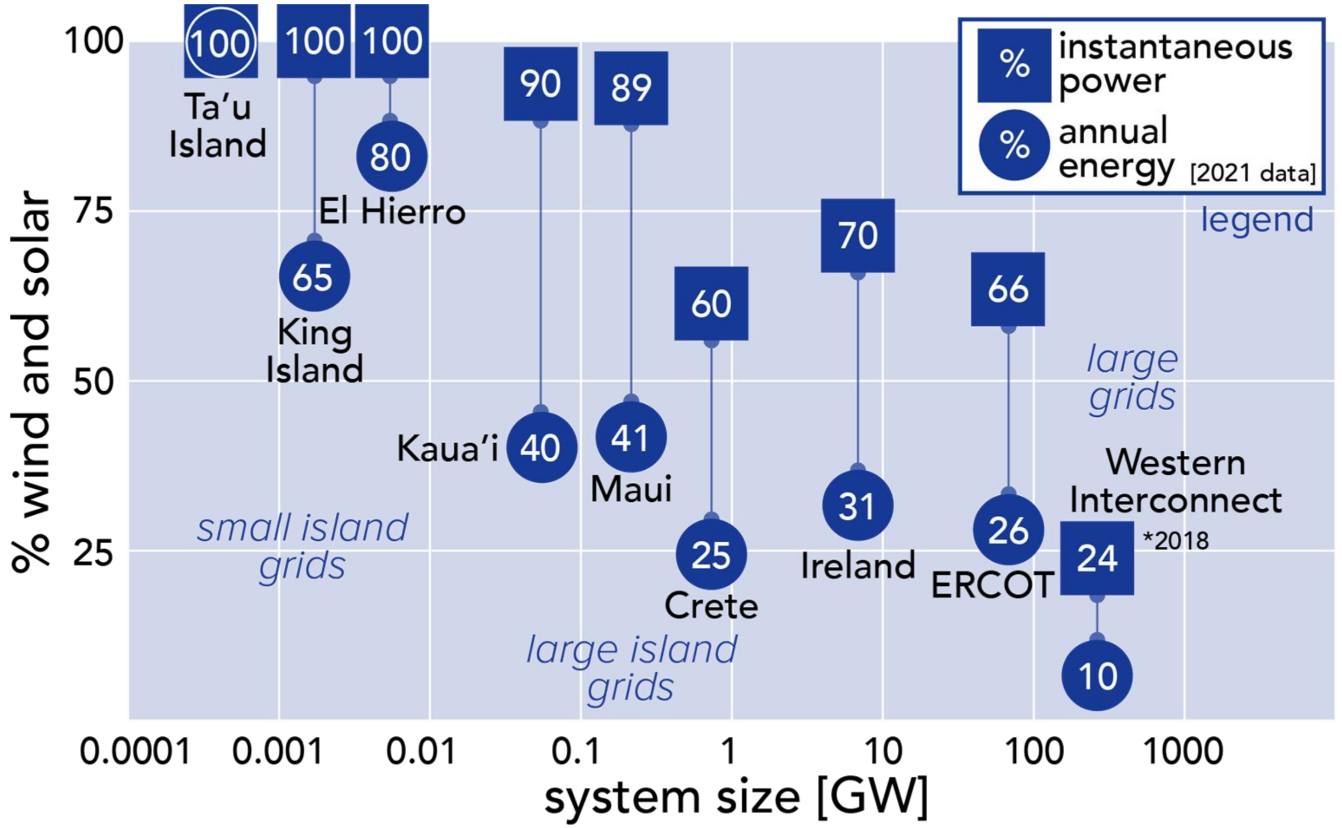
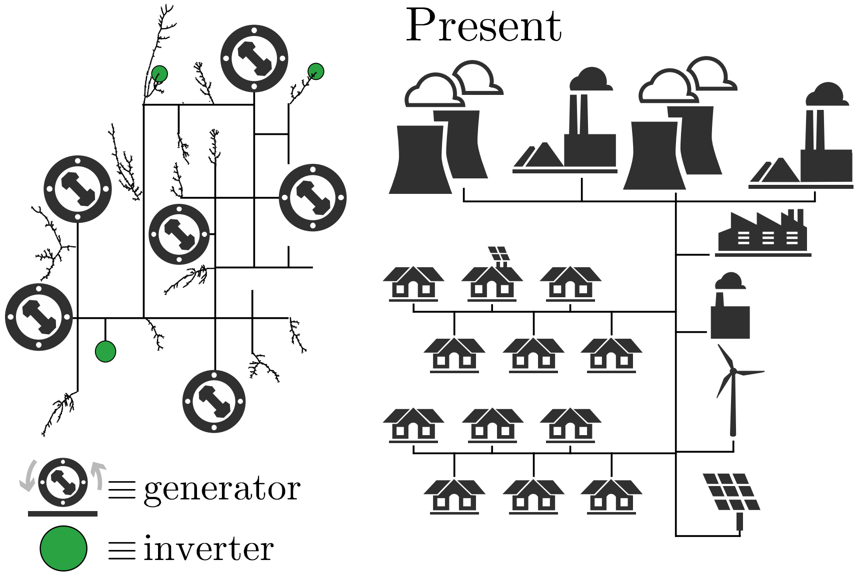
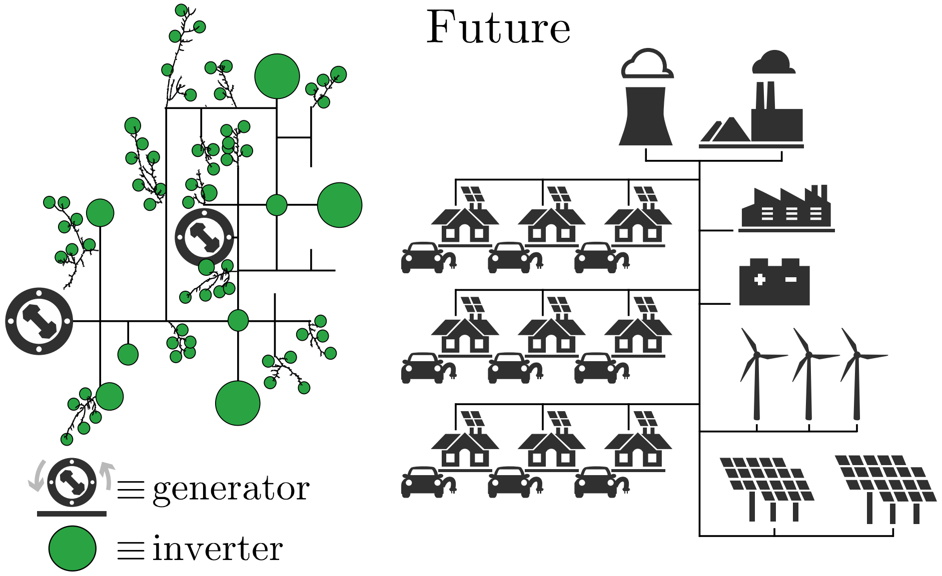
Renewable Utilization for Various Grid Sizes



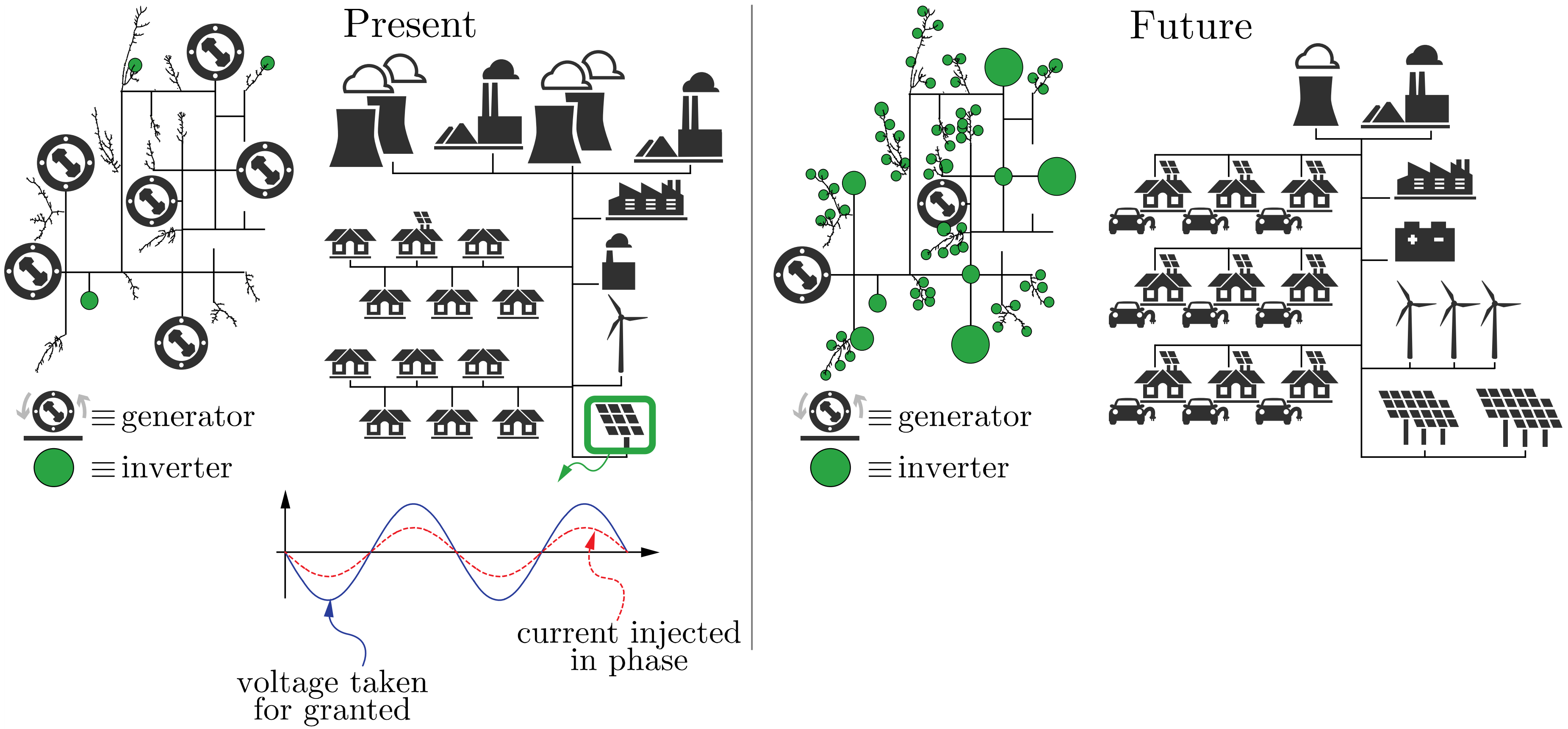
- Engineering challenges grow with system size and complexity
- Need scalable, robust, and resilient methods for system operation

A Vision for the Future Grid

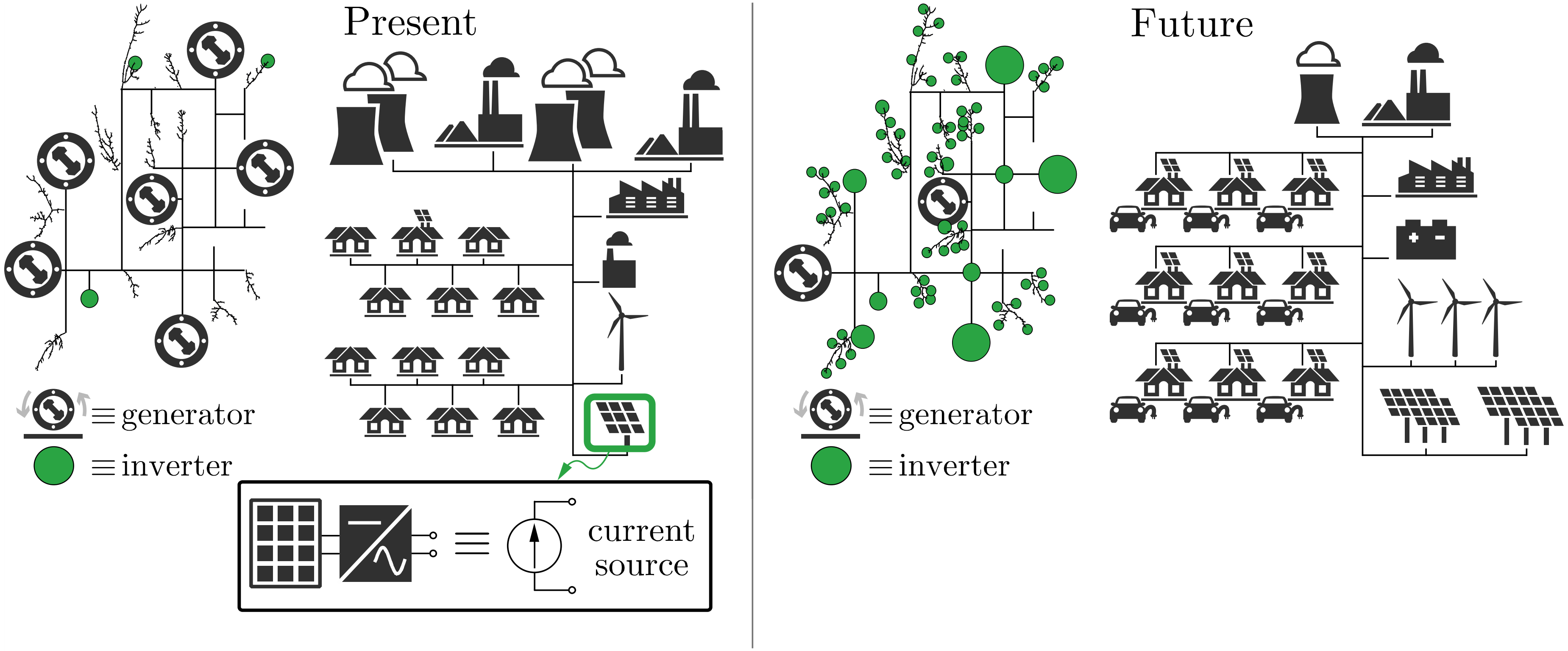




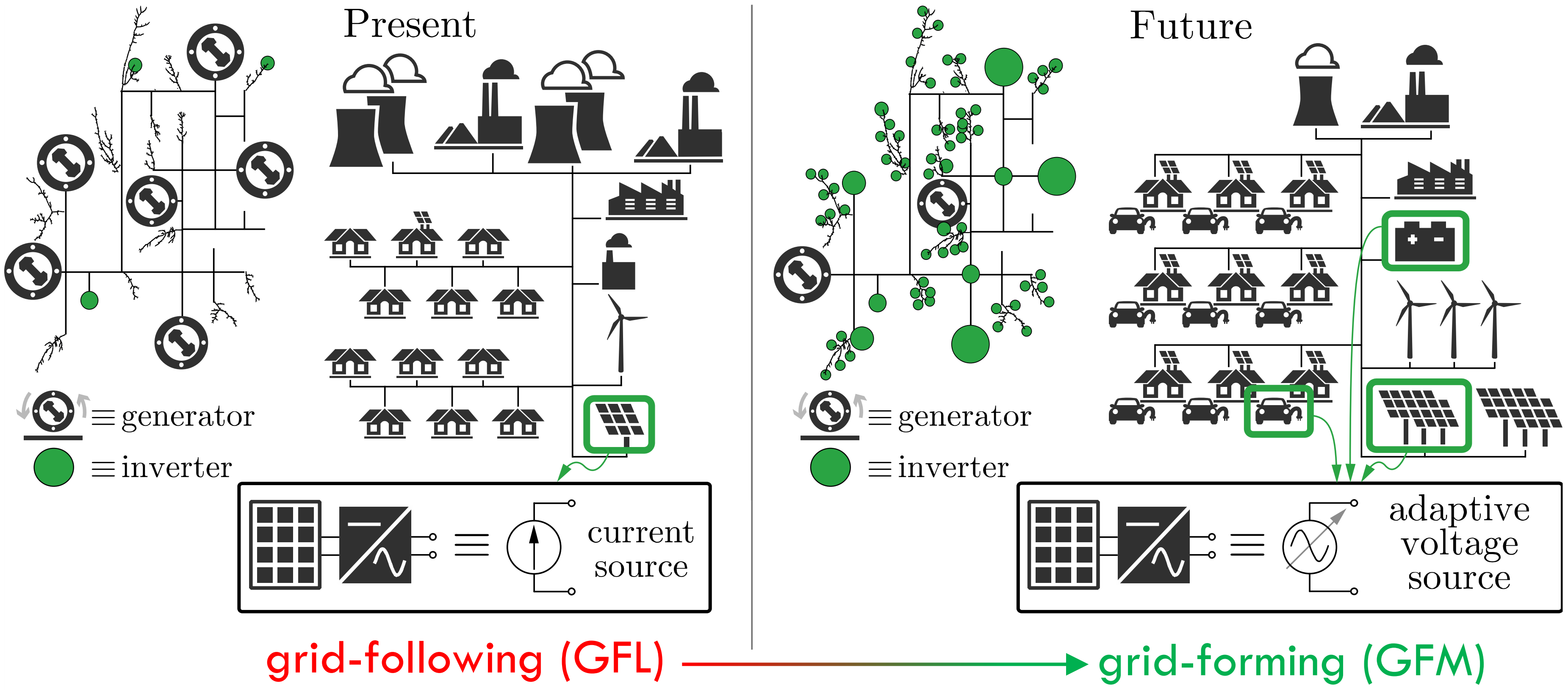
A Vision for the Future Grid



A Vision for the Future Grid



A Vision for the Future Grid

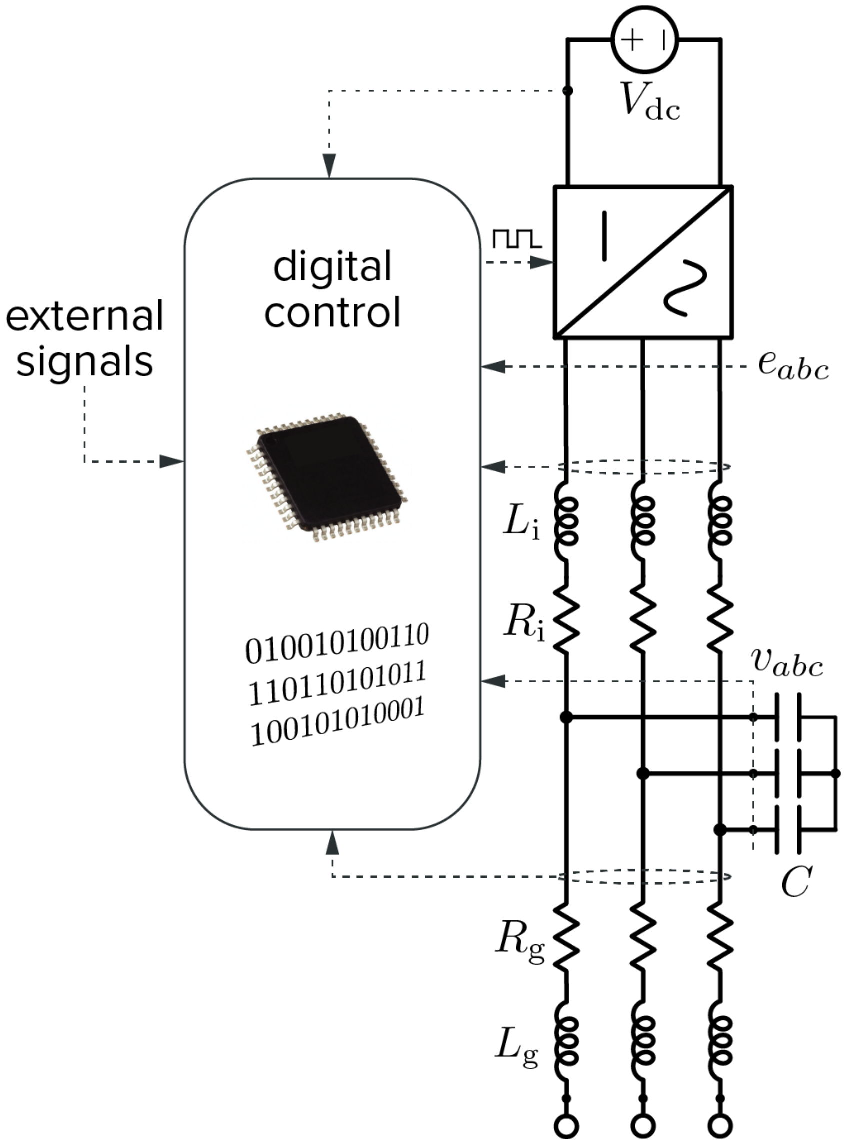


Achieving 2 goals simultaneously:

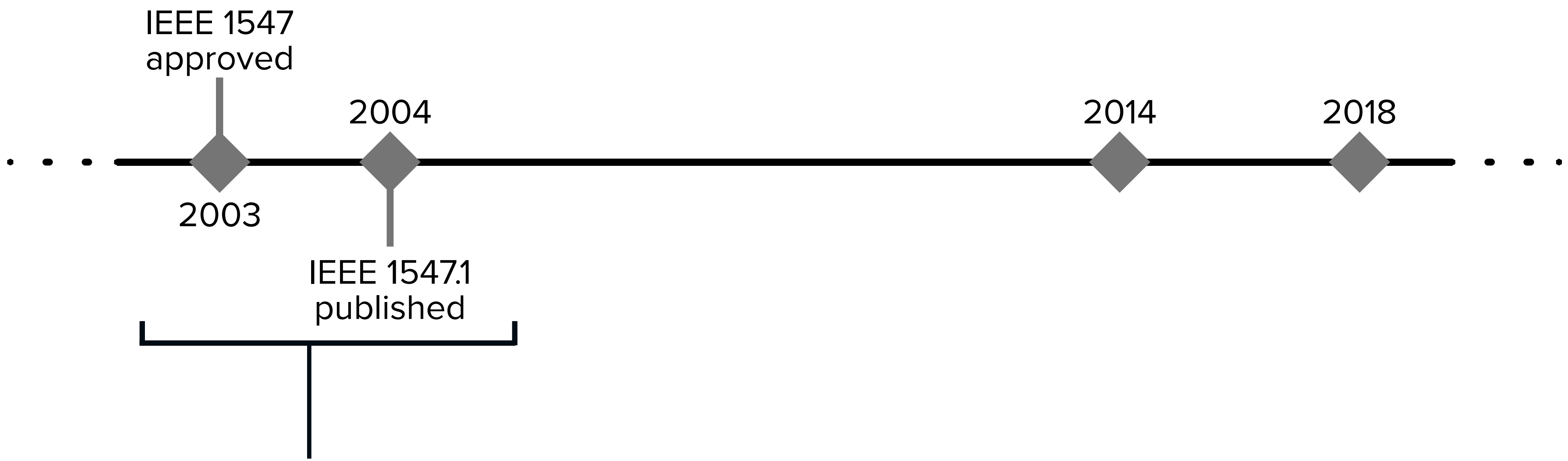
- Break down barriers that limit adoption of renewable energy
- Realize a bottom-up system that works resiliently at any scale

Desired Characteristics of a GFM Inverter

- Can operate in standalone or with many other GFMs
- Proportional power sharing among units
- Works when connected to a stiff or weak grids
- Automatic synchronization
- Exogenous signals can adjust power delivery
- Has current limiter to prevent overcurrents
- Integrates with dc-side controls



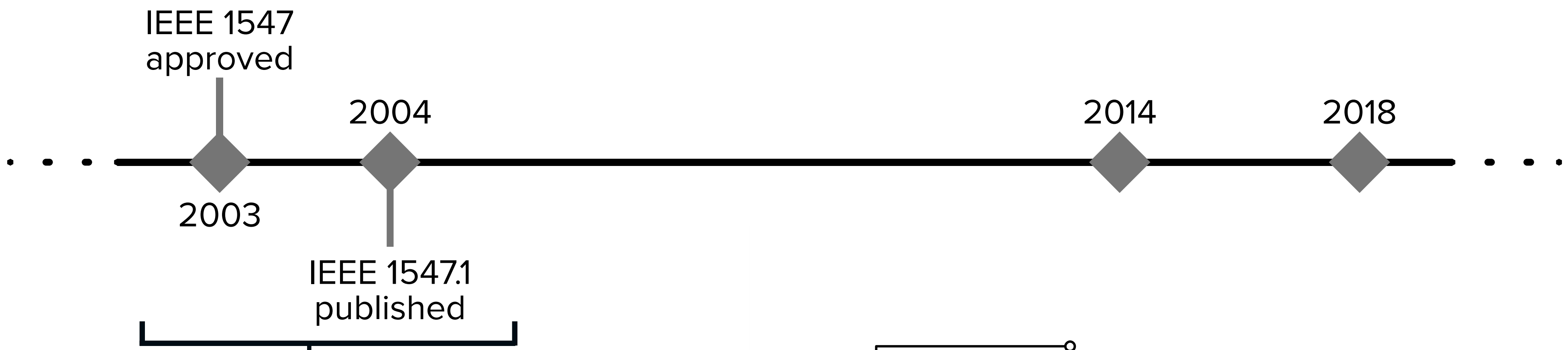
An Evolving Regulatory Landscape



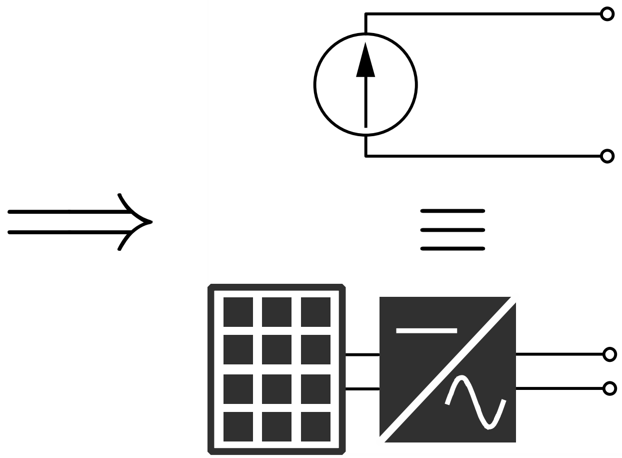
The inverter shall:

- NOT regulate voltage
- TRIP under abnormal voltage/frequency

An Evolving Regulatory Landscape



GFL unit acts as passive current injection & disconnects abruptly

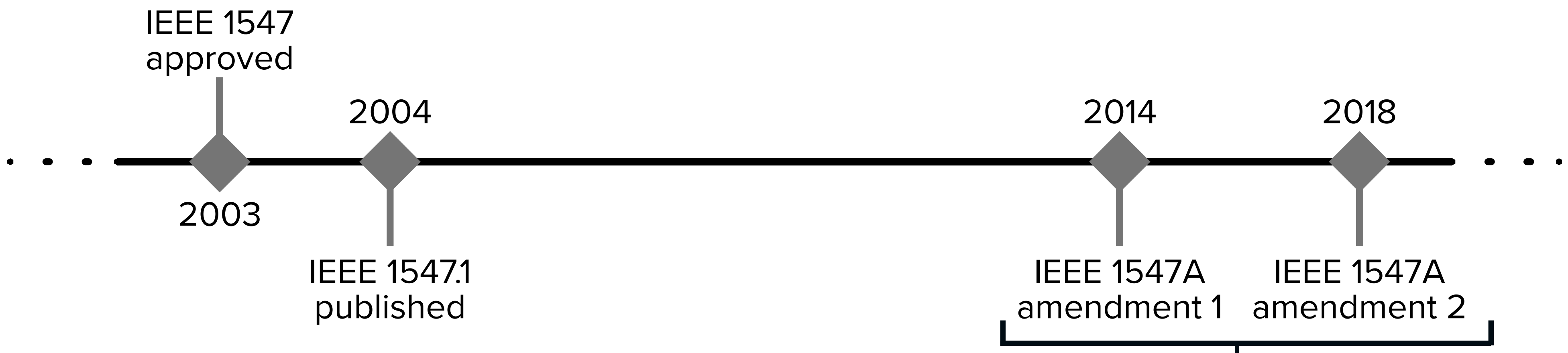


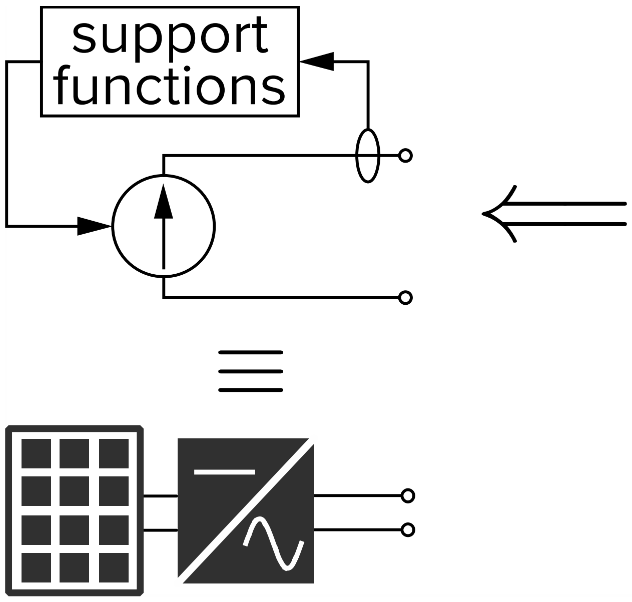
An Evolving Regulatory Landscape

The inverter shall:

- Assist voltage regulation
- Ride-through abnormal voltage/frequency

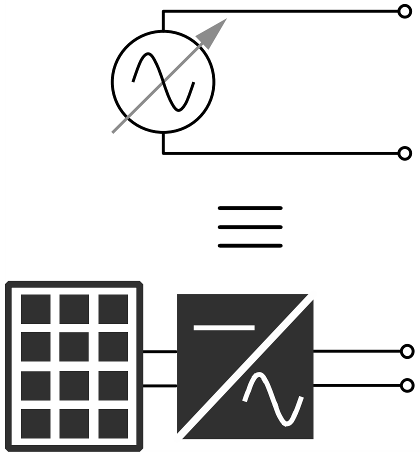
An Evolving Regulatory Landscape





GFL supplemented with functions for grid support

An Evolving Regulatory Landscape



Full-fledged grid-forming (GFM) functionality needed to realize ambitious renewable targets[1]

[1] List of territories, states, cities, communities committed to 100% renewable energy: www.sierraclub.org/ready-for-100

Remaking the Electric Power Industry

The organization that will bring GFM technologies onto grids:

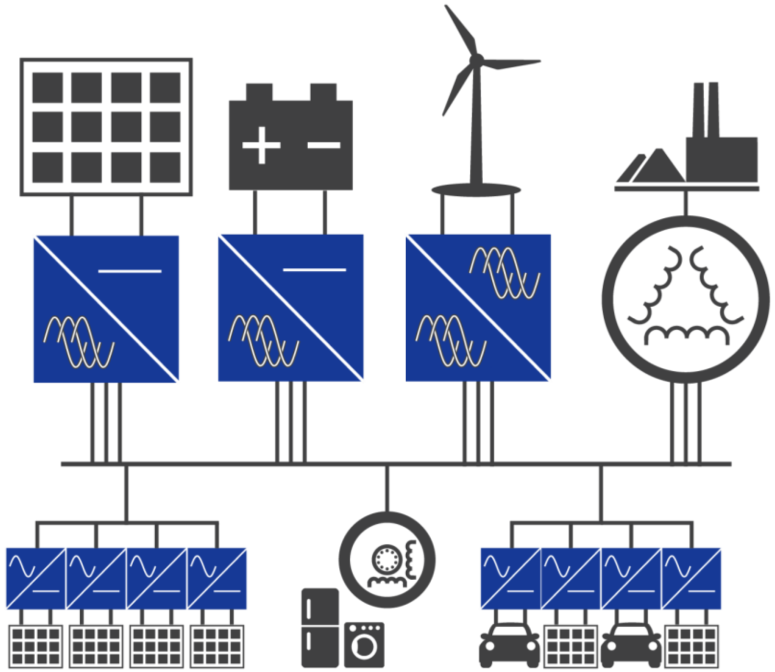
- Duration: 2022-2027
- Co-Leads: UW, NREL, EPRI
- Total Funds: $34.9M
- Federal Funds: $25M
- Membership:
  - 12 universities
  - 4 national labs
  - 25 industry members

unifi consortium
universal interoperability for grid-forming inverters

Funded by:

SOLAR ENERGY TECHNOLOGIES OFFICE
U.S. Department Of Energy

Unifying Technologies Across All Scales



unifi will unify

- New & Old (GFM with GFL+Machines)
- Local & Global (controls)
- Slow & Fast (timescales of operation)
- Big & Small (inverters to aggregations)
- Solar, Wind, & Storage (technologies)

A Team with Crosscutting Perspectives Across the Electric Power Industry

National Labs & Research Institutes

NREL Transforming ENERGY

EPRI

Sandia National Laboratories

PNNL

BERKELEY LAB

Universities

THE UNIVERSITY OF TEXAS AT AUSTIN

NC STATE UNIVERSITY

ASU

Cal

UAF

VT

unifi consortium

Industry

SIEMENS

Danfoss

SMA

ANALOG DEVICES

EATON

Ørsted

Hitachi Energy

OPAL-RT TECHNOLOGIES

Typhoon HIL

Utilities & System Operators

SOUTHERN CALIFORNIA EDISON

Hawaiian Electric

Southern Company

PG&E

Xcel ENERGY

MISO

New York Power Authority

ComEd

PGE

CAISO

PACIFICORP

ISO new england

A Team with Crosscutting Perspectives Across the Electric Power Industry

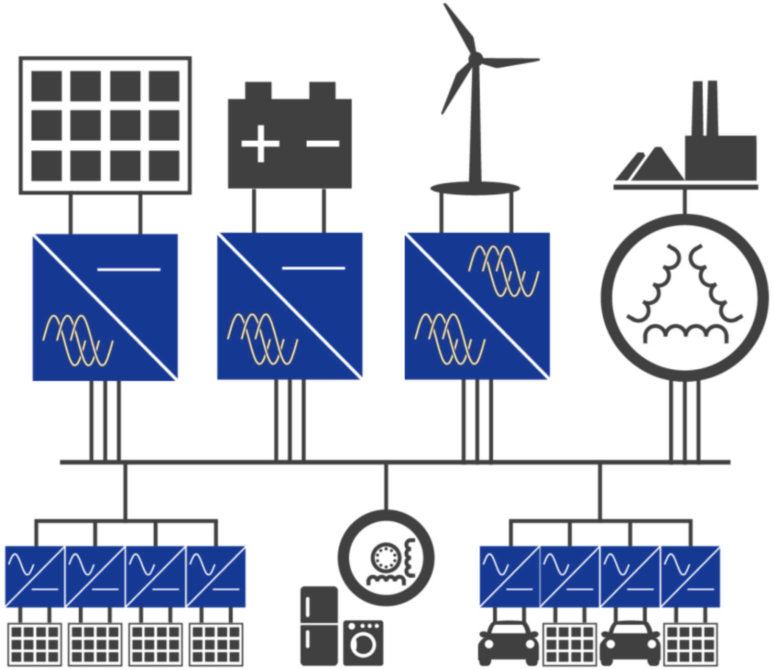
SIEMENS
Danfoss
EATON
Hitachi Energy
SMA
Ørsted

These + many others will need to work towards consensus

Universal Interoperable Operation is Essential

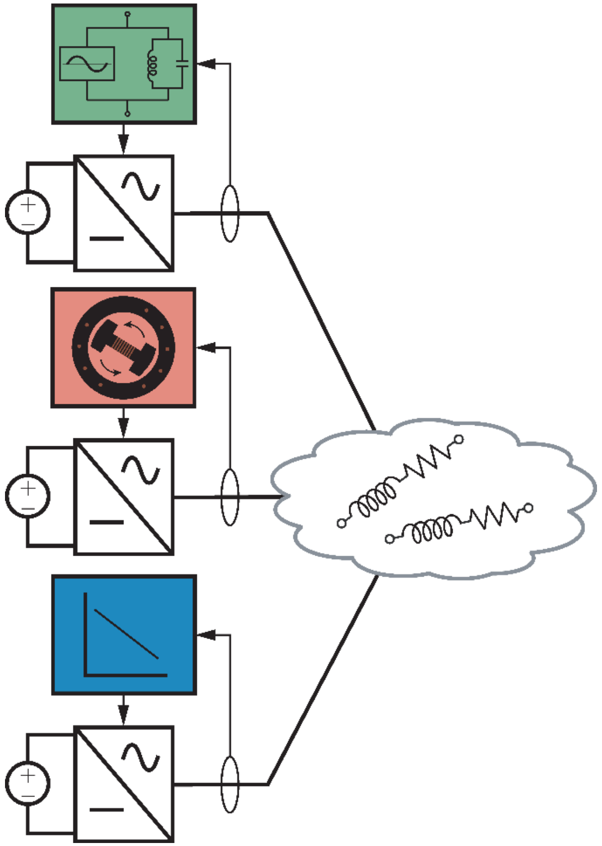
Seeking inspiration from Bluetooth®

& cultivating an ecosystem of innovation

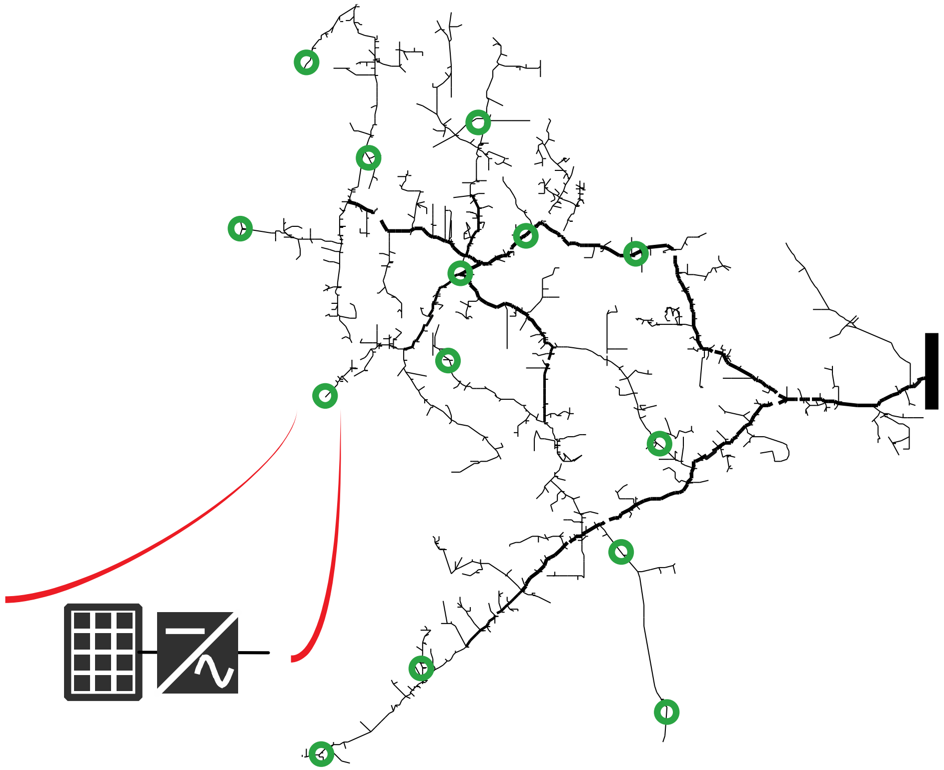


Interoperable GFM controls will

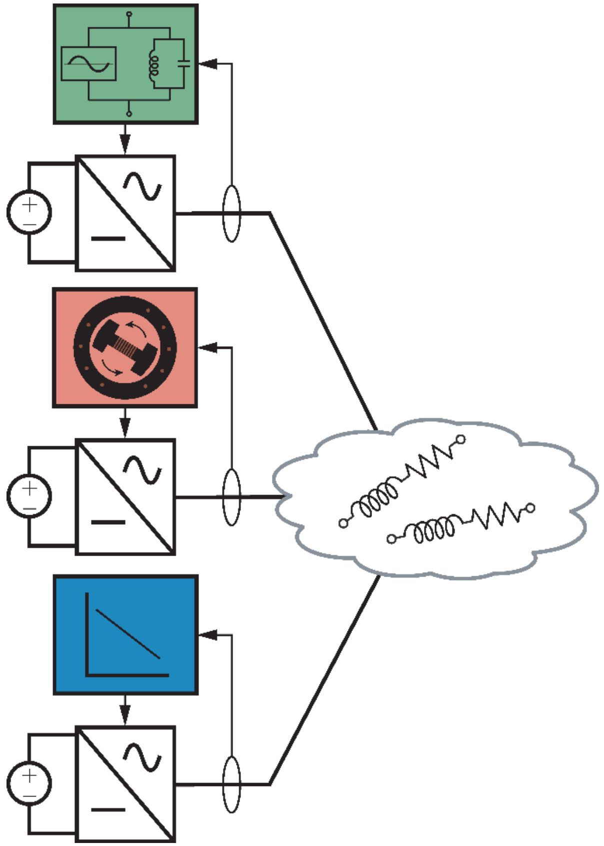
- Enable all manufacturers to innovate
- Coexist with proprietary functions
- Streamline first-principles-based standards
- Be black-box testable for certification



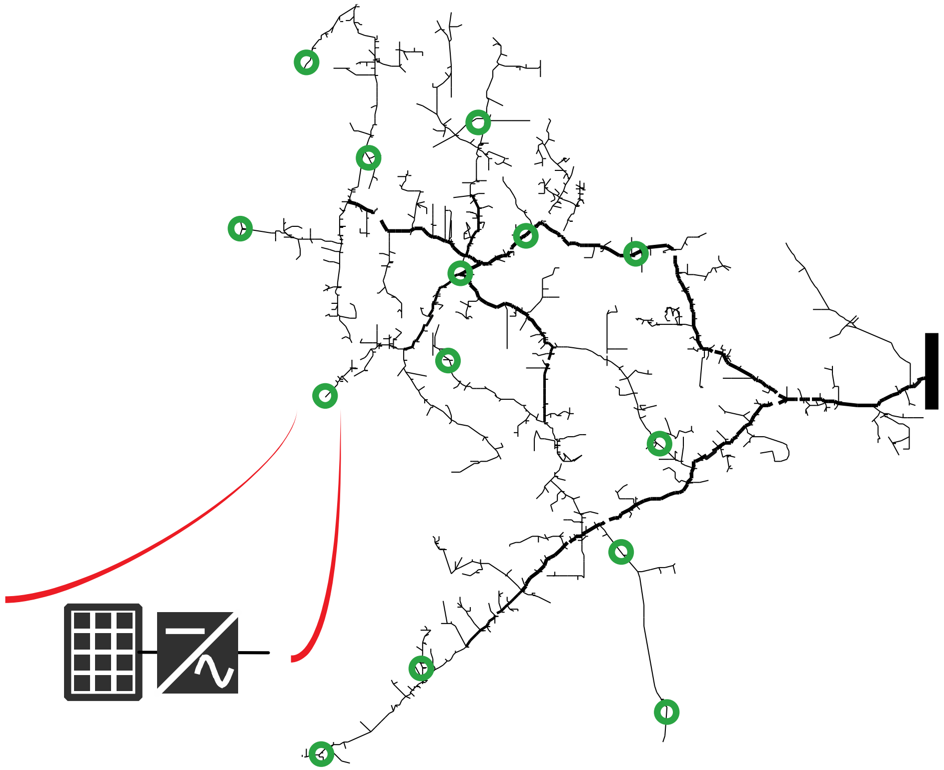
Interoperability



Self-organizing GFM Networks

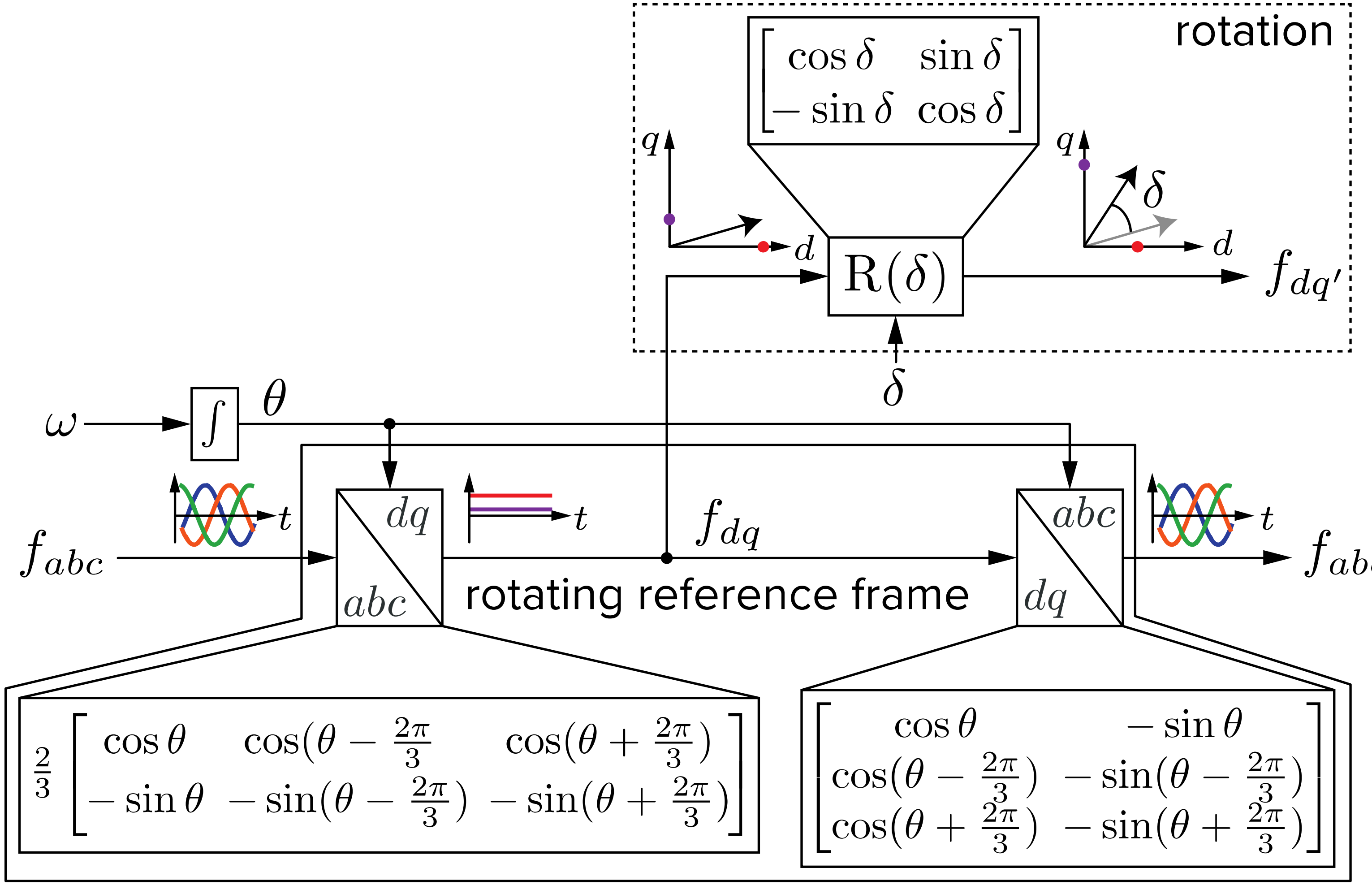


Interoperability



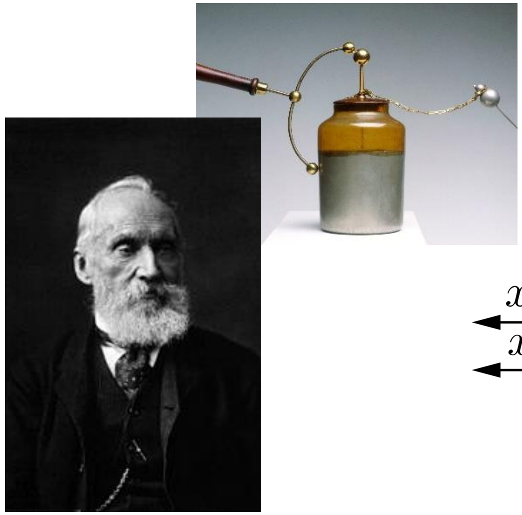
Self-organizing GFM Networks

Reference Frame Basics for Three-phase Systems



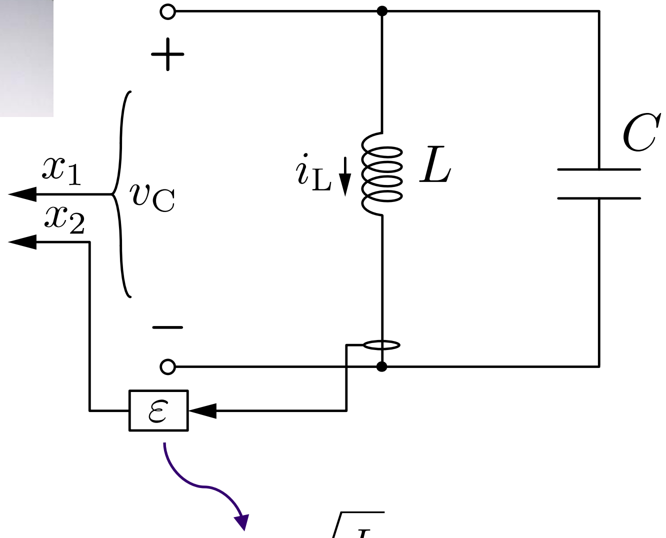
We Begin with a Humble Little Circuit

The simple harmonic oscillator lies at the heart of modern GFM control



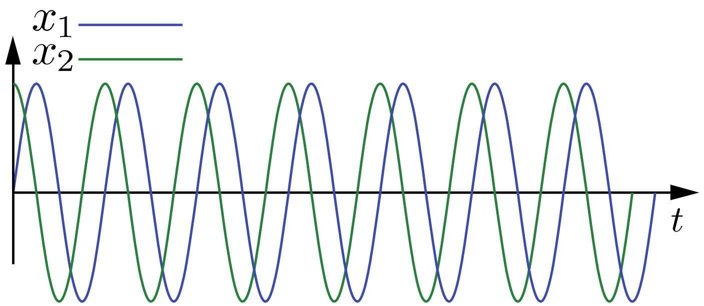
William Thomson

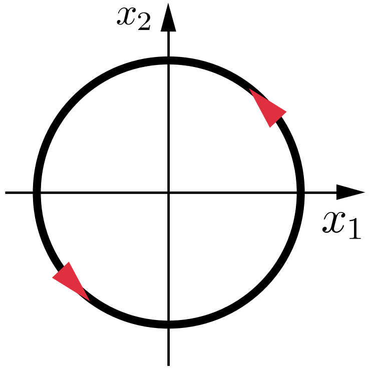
1824-1907



$$\varepsilon = \sqrt{\frac{L}{C}}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & -\omega_0 \\ \omega_0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$





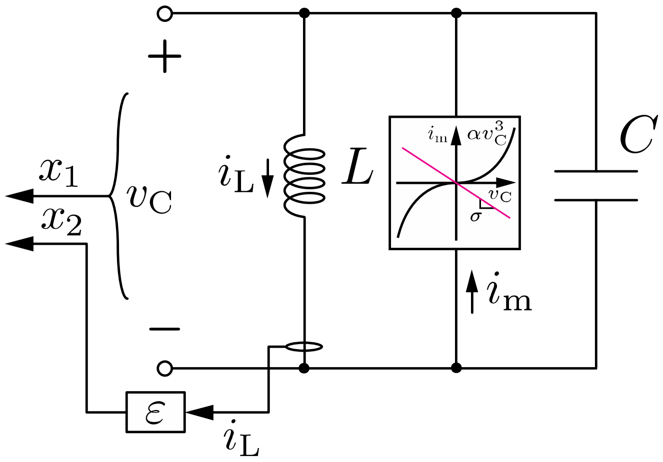
[1] Blanchard, "The history of electrical resonance," Bell System Technical Journal, 1941.

Early Oscillator Models with Self-regulating Limit Cycles

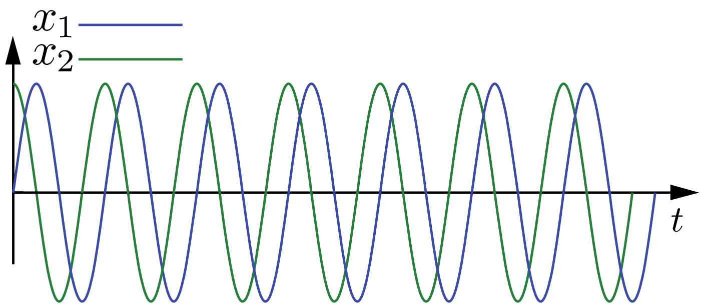
The van der Pol oscillator is a stepping-stone towards a power application

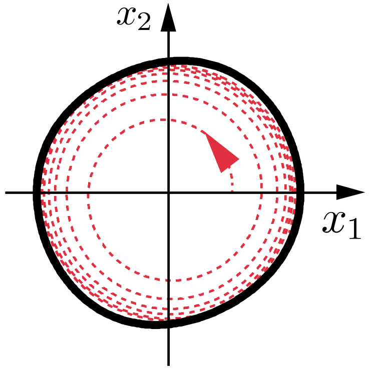
Balthasar van der Pol

1889-1959



$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} \overbrace{\boxed{\varepsilon\omega_0(\sigma - \alpha x_1^2)}}^{i_m/C} & -\omega_0 \\ \omega_0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$





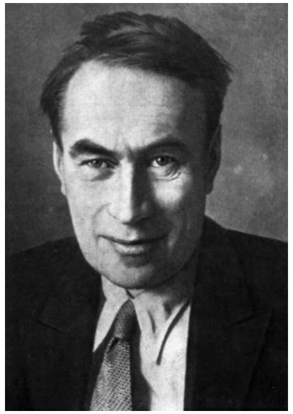
[1] van der Pol, "On "relaxation-oscillations”," The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science, 1926.

[2] B. Johnson, Dhople, Hamadeh, Krein, “Synchronization of parallel single-phase inverters using virtual oscillator control,” TPEL, 2014.

[3] Awal, Yu, Tu, Lukic, Husain, "Hierarchical control for virtual oscillator based grid-connected and islanded microgrids," TPEL, 2020.

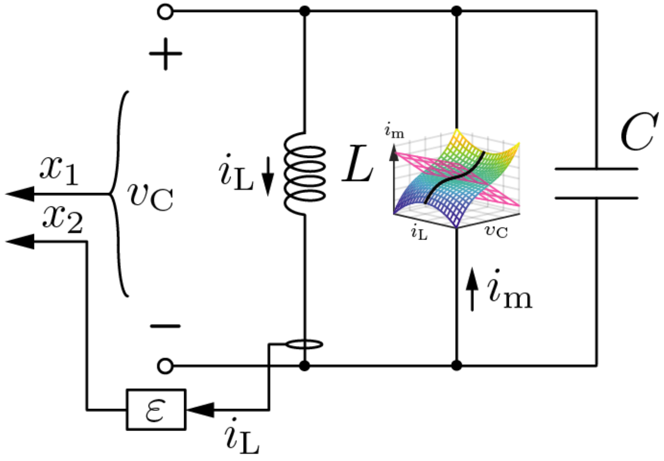
Early Oscillator Models with Self-regulating Limit Cycles

Andronov’s oscillator gives harmonic-free waveforms useful for high power quality

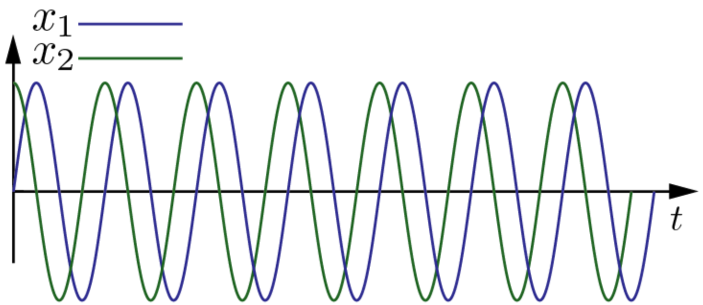


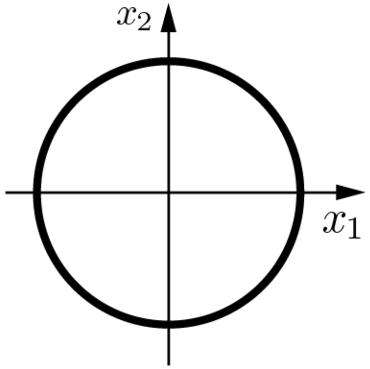
Aleksandr Andronov

1901-1952



$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix} \overbrace{\boxed{\xi(2X_{\mathrm{nom}}^2 - \|x\|^2)}}^{i_{\mathrm{m}}/C} & -\omega_0 \\ \omega_0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$





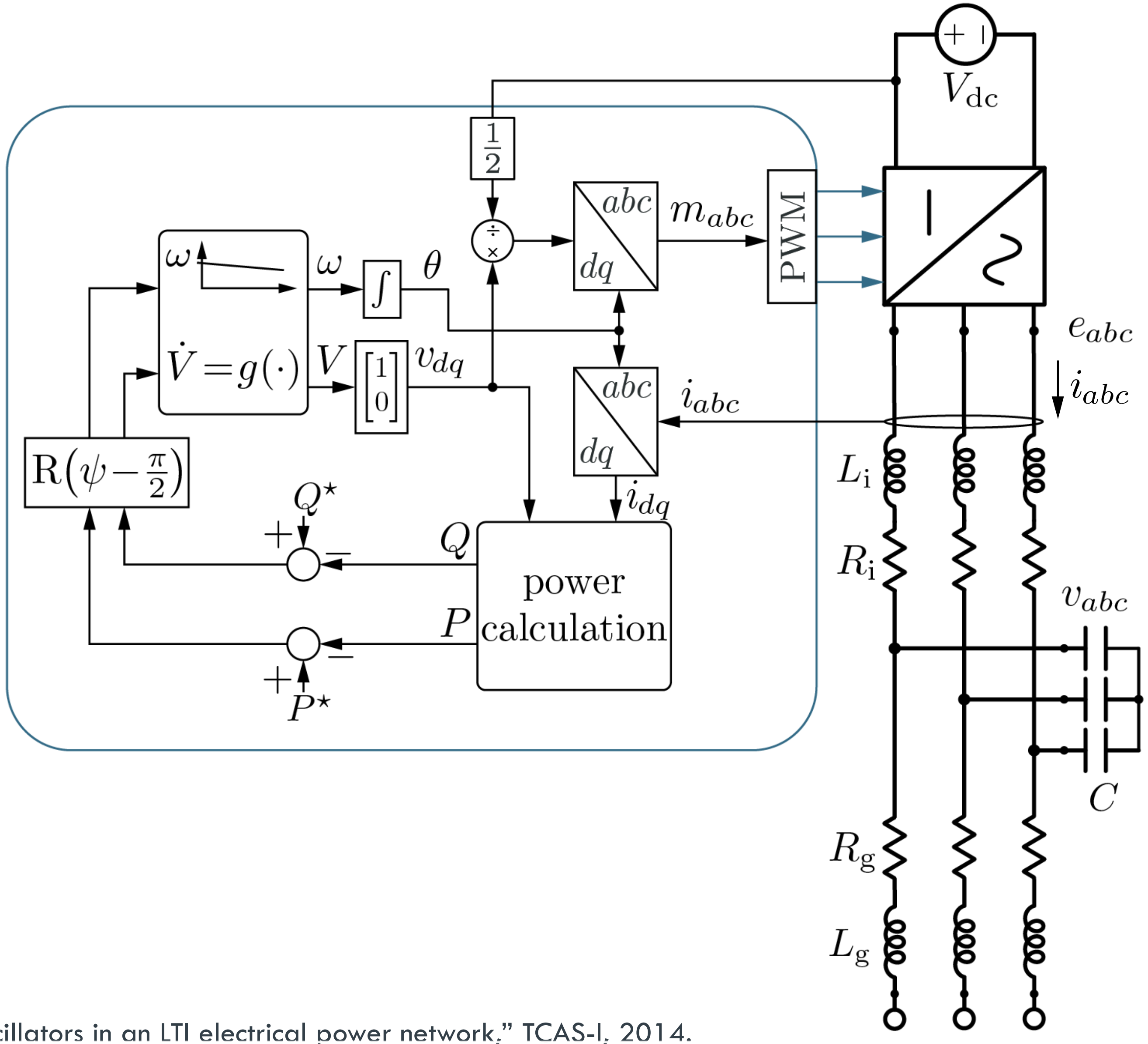
[1] Andronov, Leontovich, Gordon, Maier, “Theory of Bifurcations of Dynamical Systems on a Plane,” Israel Program Science Translation, 1971.

[2] Columbino, Gross, Dorfler, "Global phase and voltage synchronization for power inverters: A decentralized consensus-inspired approach," CDC, 2017.

[3] Lu, Dutta, Purba, Dhople, Johnson, “A grid-compatible virtual oscillator controller: Analysis and design” ECCE, 2019.

Approach #1: Dispatchable Virtual Oscillator Control

- Transformed oscillator gives model here
- Newest GFM type in existence



[1] Johnson, Dhople, Hamadeh, Krein, "Synchronization of nonlinear oscillators in an LTI electrical power network," TCAS-I, 2014.

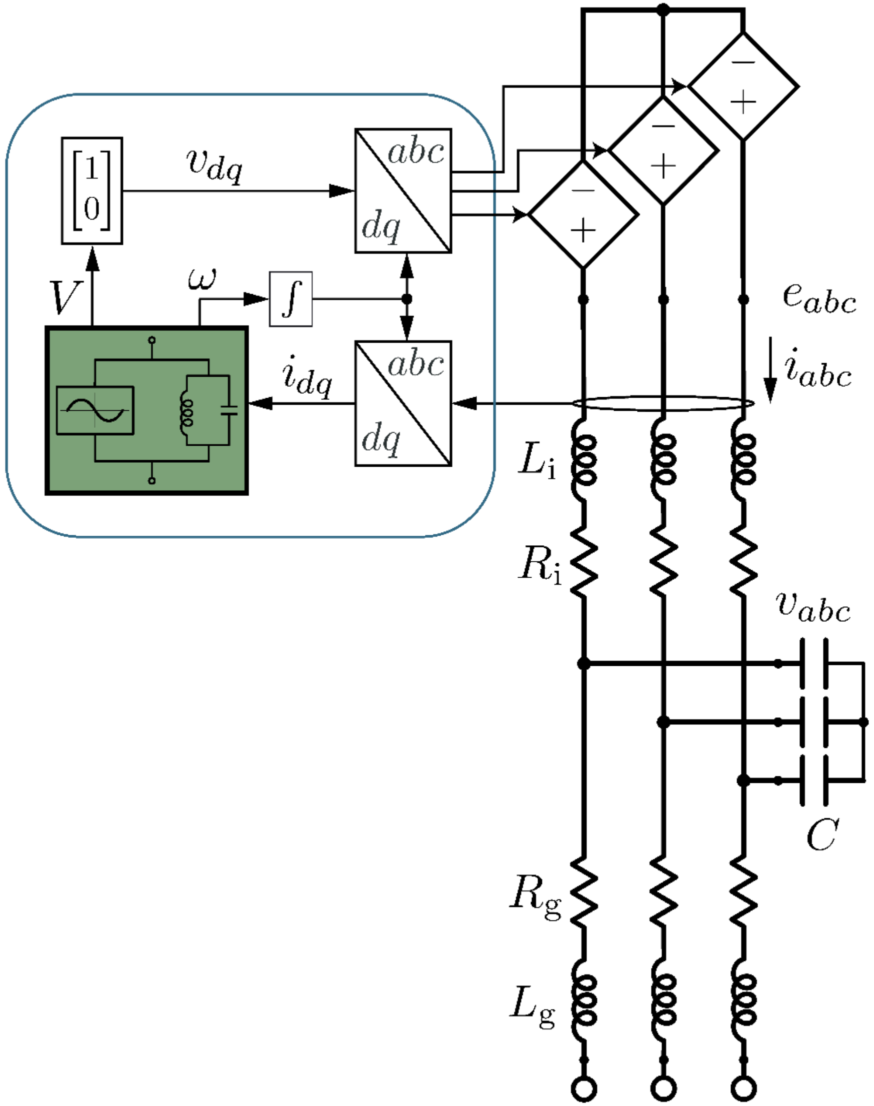
[2] Colombino, Groß, Dorfler, "Global phase and voltage synchronization for power inverters: A decentralized consensus-inspired approach," CDC, 2017.

Approach #1: Dispatchable Virtual Oscillator Control

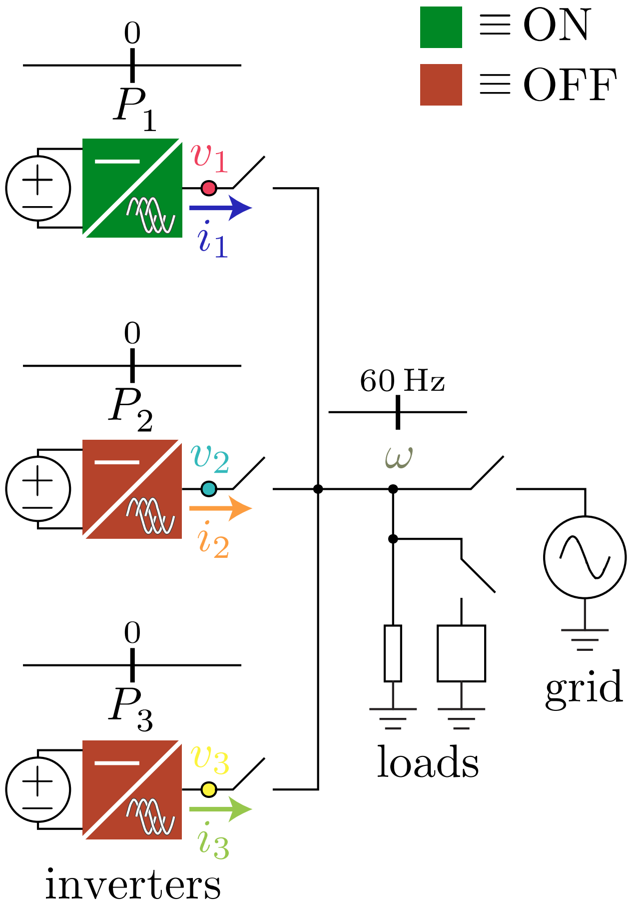
Control equations are given by

$$\omega = \omega_0 + \frac{\omega_0 \kappa_1}{V^2} \begin{bmatrix} 1 & 0 \end{bmatrix} \mathrm{R}(\psi - \tfrac{\pi}{2}) \begin{bmatrix} P^\star - P \\ Q^\star - Q \end{bmatrix},$$

$$\dot{V} = \omega_0 \kappa_2 V (V_0^2 - V^2) + \frac{\omega_0 \kappa_1}{V} \begin{bmatrix} 0 & 1 \end{bmatrix} \mathrm{R}(\psi - \tfrac{\pi}{2}) \begin{bmatrix} P^\star - P \\ Q^\star - Q \end{bmatrix}.$$

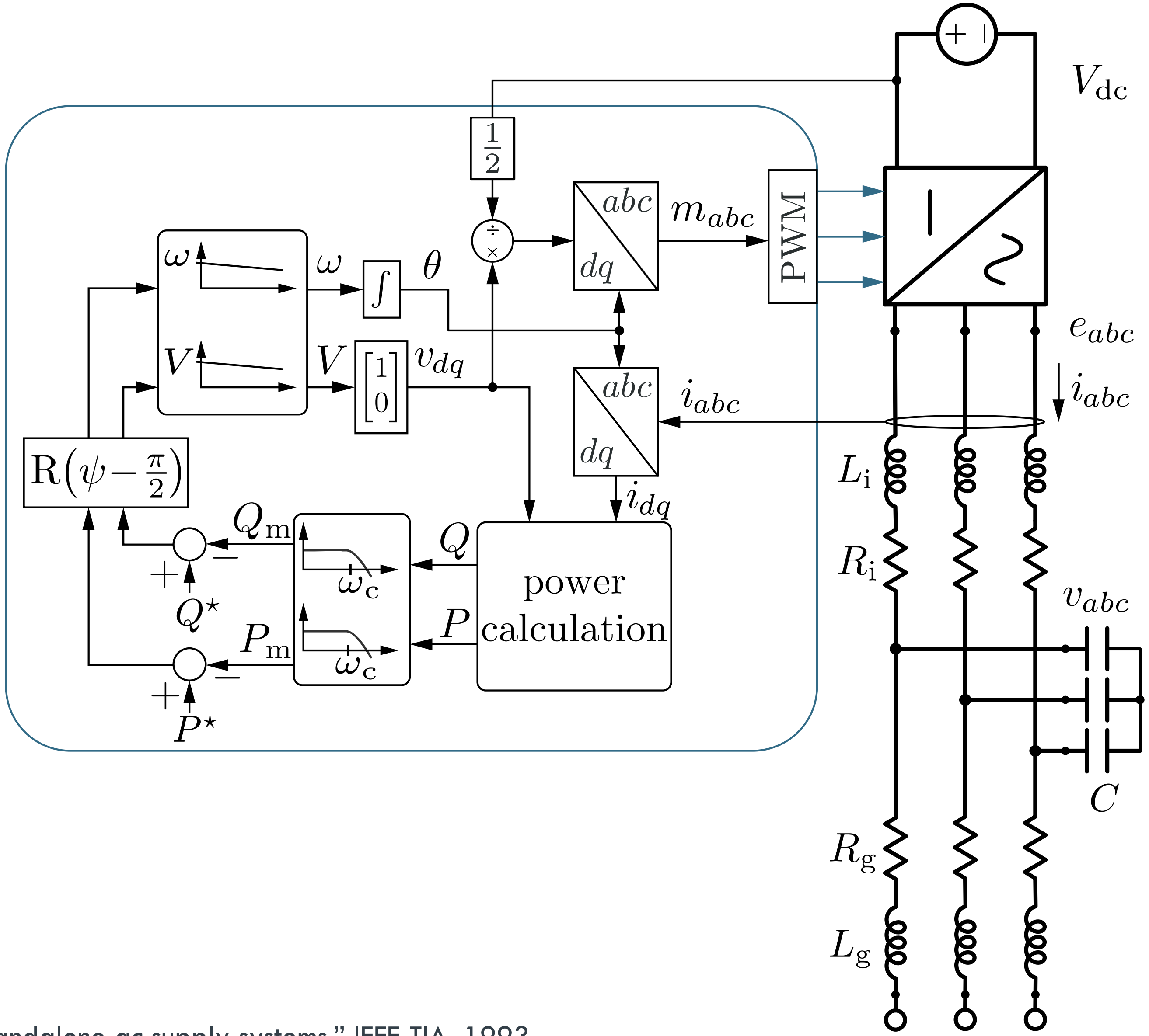


Illustrating Versatile Performance on Hardware with dVOC Control



Approach #2: Droop Control

- Oldest GFM method in existence
- Inspired by machine droop laws



[1] M. Chandorkar, et. al, “Control of parallel connected inverters in standalone ac supply systems,” IEEE TIA, 1993

[2] J. Guerrero, et. al, “Hierarchical control of droop-controlled dc and ac microgrids,” IEEE TIE, 2011.

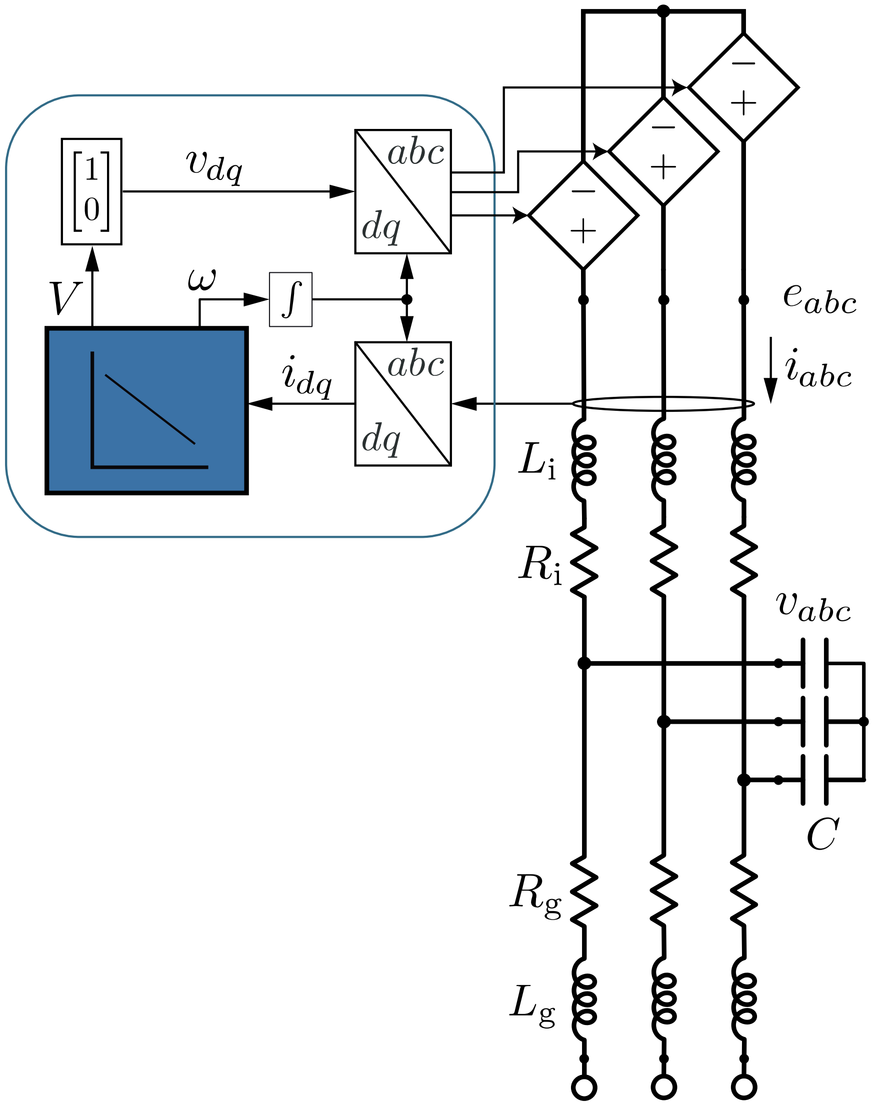
Approach #2: Droop Control

Control equations are given by

$$\omega = \omega_0 + \tfrac{1}{d_{\mathrm{f}}}\begin{bmatrix}1 & 0\end{bmatrix}\mathrm{R}(\psi - \tfrac{\pi}{2})\begin{bmatrix}P^\star - P_{\mathrm{m}}\\ Q^\star - Q_{\mathrm{m}}\end{bmatrix},$$

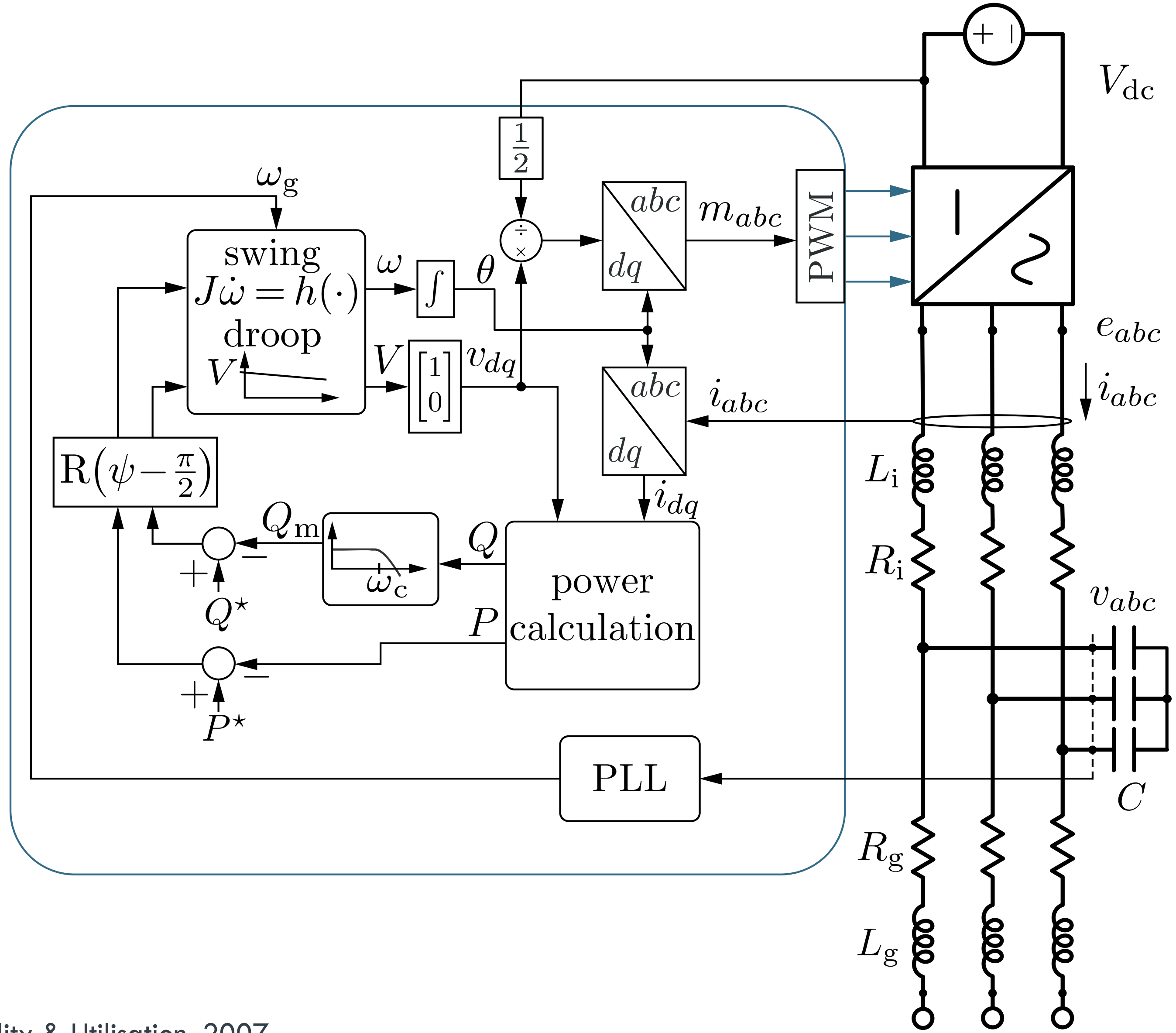
$$V = V_0 + \tfrac{1}{d_{\mathrm{v}}}\begin{bmatrix}0 & 1\end{bmatrix}\mathrm{R}(\psi - \tfrac{\pi}{2})\begin{bmatrix}P^\star - P_{\mathrm{m}}\\ Q^\star - Q_{\mathrm{m}}\end{bmatrix},$$

$$\frac{1}{\omega_{\mathrm{c}}}\begin{bmatrix}\dot{P}_{\mathrm{m}}\\ \dot{Q}_{\mathrm{m}}\end{bmatrix} = -\begin{bmatrix}P_{\mathrm{m}}\\ Q_{\mathrm{m}}\end{bmatrix} + \begin{bmatrix}P\\ Q\end{bmatrix}.$$



Approach #3: Virtual Synchronous Machine Control

- Emulate machine dynamics digitally
- Popular due to familiar behavior

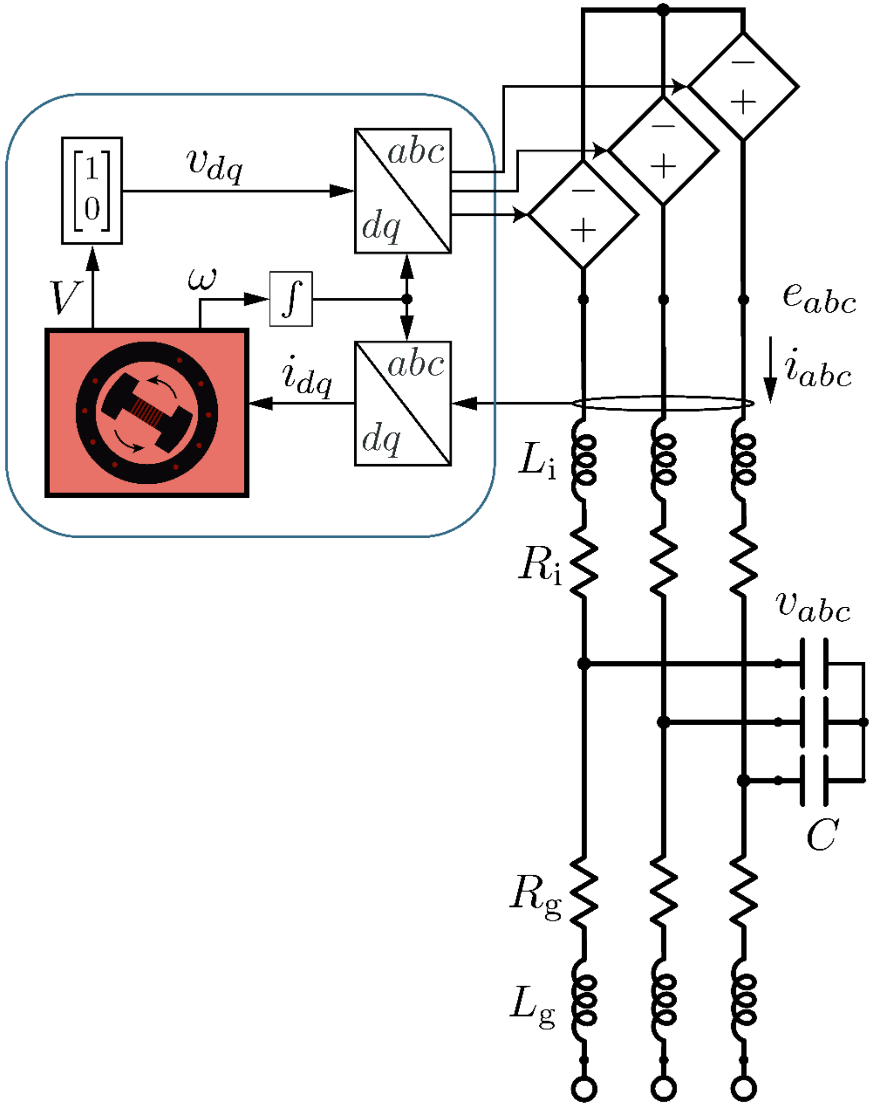


[1] Beck, Hesse, "Virtual synchronous machine," Electrical Power Quality & Utilisation, 2007.
[2] D'Arco, Suul, Fosso, "A virtual synchronous machine implementation for distributed control of power converters in smartgrids," EPSR, 2014.

Approach #3: Virtual Synchronous Machine Control

Control equations are given by

$$
\begin{aligned}
J\dot{\omega} &= -\omega + \omega_0 + \frac{d_\mathrm{d}}{d_\mathrm{f}}(\omega_\mathrm{g} - \omega) \\
&\quad + \frac{1}{d_\mathrm{f}}\begin{bmatrix}1 & 0\end{bmatrix}\mathrm{R}(\psi - \tfrac{\pi}{2})\begin{bmatrix}P^\star - P \\ Q^\star - Q_\mathrm{m}\end{bmatrix}, \\
V &= V_0 + \frac{1}{d_\mathrm{v}}\begin{bmatrix}0 & 1\end{bmatrix}\mathrm{R}(\psi - \tfrac{\pi}{2})\begin{bmatrix}P^\star - P \\ Q^\star - Q_\mathrm{m}\end{bmatrix}, \\
\frac{1}{\omega_0}\dot{\eta} &= \begin{bmatrix}0 & 1\end{bmatrix}\mathrm{R}(\alpha)\mathrm{R}(\delta)\mathrm{T}(\omega_0 t)v, \\
\frac{1}{\omega_0}\dot{\alpha} &= \frac{k_\mathrm{P}}{\omega_0}\dot{\eta} + k_\mathrm{I}\eta, \\
\frac{1}{\omega_\mathrm{c}}\dot{Q}_\mathrm{m} &= -Q_\mathrm{m} + Q\ .
\end{aligned}
$$



A Universal & Unified GFM Model

All 3 GFM types can be boiled down to

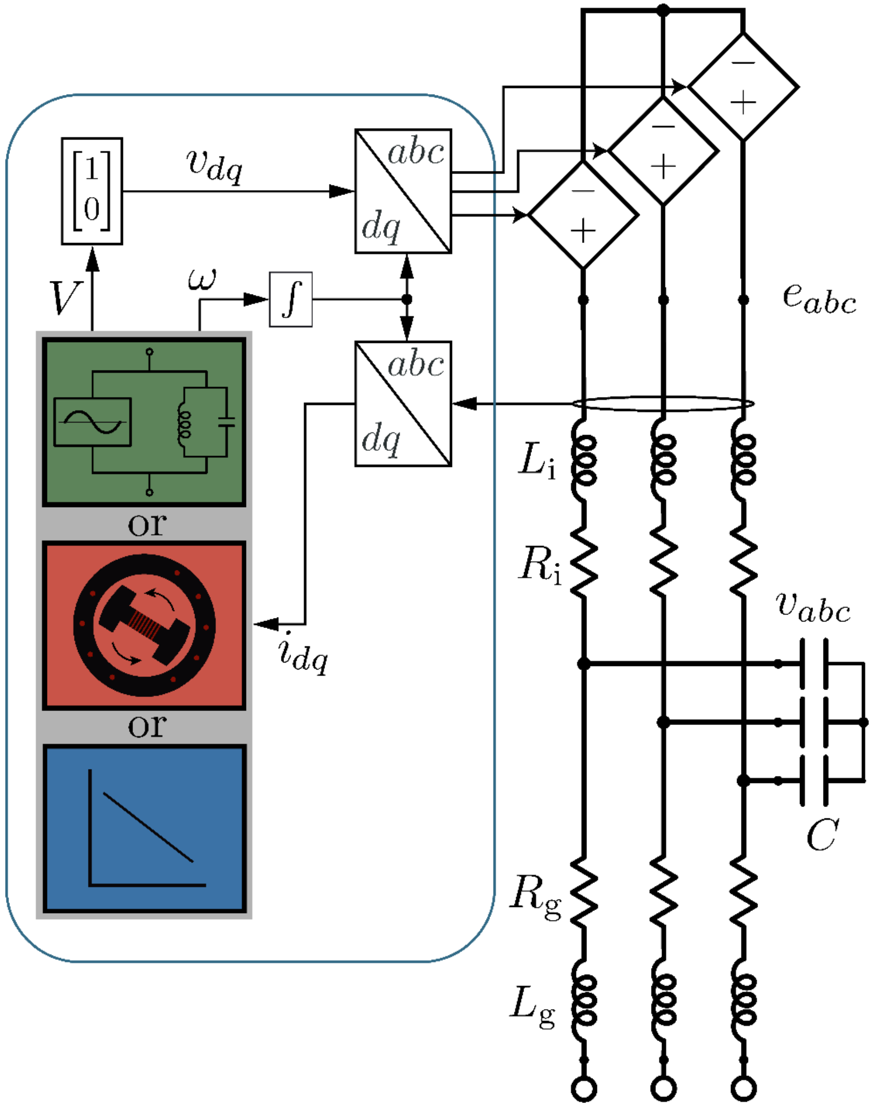
$$\tau_{\mathrm{f}}\frac{\mathrm{d}\omega}{\mathrm{d}t} = -\omega + \omega_0 + \kappa_{\mathrm{d}}\big(\omega_{\mathrm{g}} - \omega\big) + \kappa_{\mathrm{f}}\begin{bmatrix}1 & 0\end{bmatrix}\mathrm{R}(\psi - \tfrac{\pi}{2})\begin{bmatrix}P^\star - P_{\mathrm{m}} \\ Q^\star - Q_{\mathrm{m}}\end{bmatrix},$$

$$\tau_{\mathrm{v}}\frac{\mathrm{d}V}{\mathrm{d}t} = f_{\mathrm{v}}(V) + \kappa_{\mathrm{v}}\begin{bmatrix}0 & 1\end{bmatrix}\mathrm{R}(\psi - \tfrac{\pi}{2})\begin{bmatrix}p^\star - p_{\mathrm{m}} \\ q^\star - q_{\mathrm{m}}\end{bmatrix},$$

$$\frac{1}{\omega_0}\frac{\mathrm{d}\eta}{\mathrm{d}t} = \begin{bmatrix}0 & 1\end{bmatrix}\mathrm{R}(\alpha)\mathrm{R}(\delta)\mathrm{T}(\omega_0 t)v,$$

$$\frac{1}{\omega_0}\frac{\mathrm{d}\alpha}{\mathrm{d}t} = \frac{k_{\mathrm{P}}}{\omega_0}\dot{\eta} + k_{\mathrm{I}}\eta,$$

$$\tau_{\mathrm{p}}\begin{bmatrix}\dot{P}_{\mathrm{m}} \\ \dot{Q}_{\mathrm{m}}\end{bmatrix} = -\begin{bmatrix}P_{\mathrm{m}} \\ Q_{\mathrm{m}}\end{bmatrix} + \begin{bmatrix}P \\ Q\end{bmatrix}.$$



[1] Johnson, et. al, "A generic primary-control model for grid-forming inverters: Towards interoperable operation & control," HICSS, 2021.

[2] Ajala, Baeckeland, Dhople, Dominguez-Garcia, "Uncovering the Kuramoto model from full-order models of grid-forming inverter-based power networks," CDC, 2021.

A Universal & Unified GFM Model

All 3 GFM types can be boiled down to

$$\tau_{\mathrm{f}} \frac{\mathrm{d}\omega}{\mathrm{d}t} = -\omega + \omega_0 + \kappa_{\mathrm{d}}\big(\omega_{\mathrm{g}} - \omega\big) + \kappa_{\mathrm{f}} \begin{bmatrix}1 & 0\end{bmatrix} \mathrm{R}(\psi - \tfrac{\pi}{2}) \begin{bmatrix} P^\star - P_{\mathrm{m}} \\ Q^\star - Q_{\mathrm{m}} \end{bmatrix},$$

$$\tau_{\mathrm{v}} \frac{\mathrm{d}V}{\mathrm{d}t} = f_{\mathrm{v}}(V) + \kappa_{\mathrm{v}} \begin{bmatrix}0 & 1\end{bmatrix} \mathrm{R}(\psi - \tfrac{\pi}{2}) \begin{bmatrix} p^\star - p_{\mathrm{m}} \\ q^\star - q_{\mathrm{m}} \end{bmatrix},$$

$$\frac{1}{\omega_0}\frac{\mathrm{d}\eta}{\mathrm{d}t} = \begin{bmatrix}0 & 1\end{bmatrix} \mathrm{R}(\alpha)\mathrm{R}(\delta)\mathrm{T}(\omega_0 t)v,$$

$$\frac{1}{\omega_0}\frac{\mathrm{d}\alpha}{\mathrm{d}t} = \frac{k_{\mathrm{P}}}{\omega_0}\dot{\eta} + k_{\mathrm{I}}\eta,$$

$$\tau_{\mathrm{p}} \begin{bmatrix} \dot{P}_{\mathrm{m}} \\ \dot{Q}_{\mathrm{m}} \end{bmatrix} = -\begin{bmatrix} P_{\mathrm{m}} \\ Q_{\mathrm{m}} \end{bmatrix} + \begin{bmatrix} P \\ Q \end{bmatrix}.$$

where the parameters are

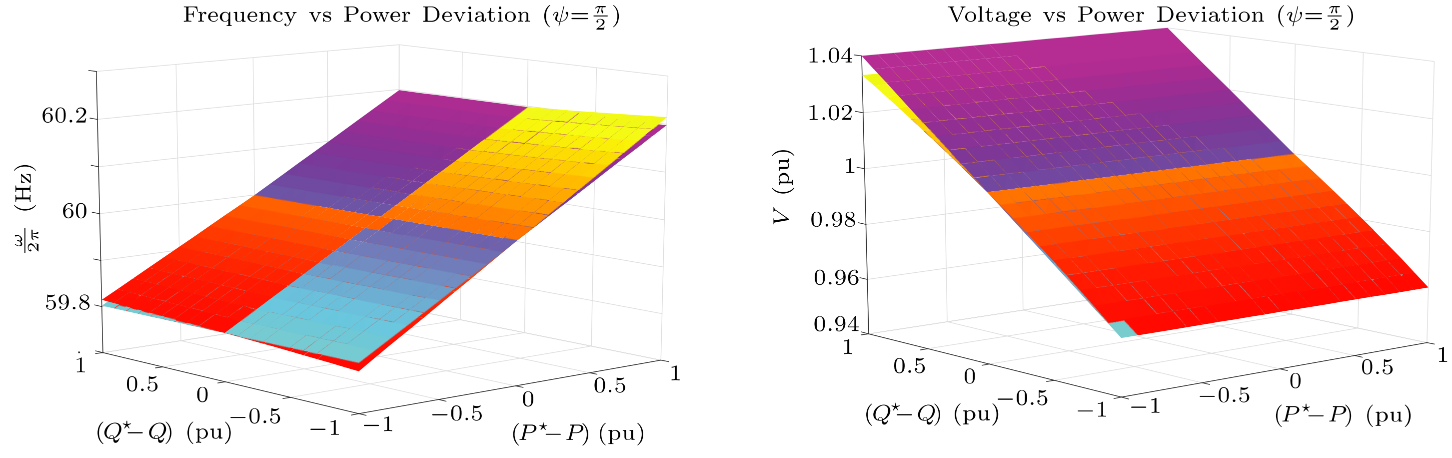
	τ_{f}	τ_{v}	τ_{p}	κ_{d}	κ_{f}	κ_{v}	$f_{\mathrm{v}}(e^\star)$
droop	0	0	$\frac{1}{\omega_{\mathrm{c}}}$	0	$\frac{1}{d_{\mathrm{f}}}$	$\frac{1}{d_{\mathrm{v}}}$	$-V + V_0$
VSM	J	0	$\frac{1}{\omega_{\mathrm{c}}}$	$\frac{d_{\mathrm{d}}}{d_{\mathrm{f}}}$	$\frac{1}{d_{\mathrm{f}}}$	$\frac{1}{d_{\mathrm{v}}}$	$-V + V_0$
dVOC	0	$\frac{1}{\omega_0}$	0	0	$\frac{\omega_0 \kappa_1}{V^2}$	$\frac{\kappa_1}{V}$	$\kappa_2(-V^2 + V_0^2)V$

[1] Johnson, et. al, "A generic primary-control model for grid-forming inverters: Towards interoperable operation & control," HICSS, 2021.
[2] Ajala, Baeckeland, Dhople, Dominguez-Garcia, "Uncovering the Kuramoto model from full-order models of grid-forming inverter-based power networks," CDC, 2021.

Steady-state Performance of GFM Types

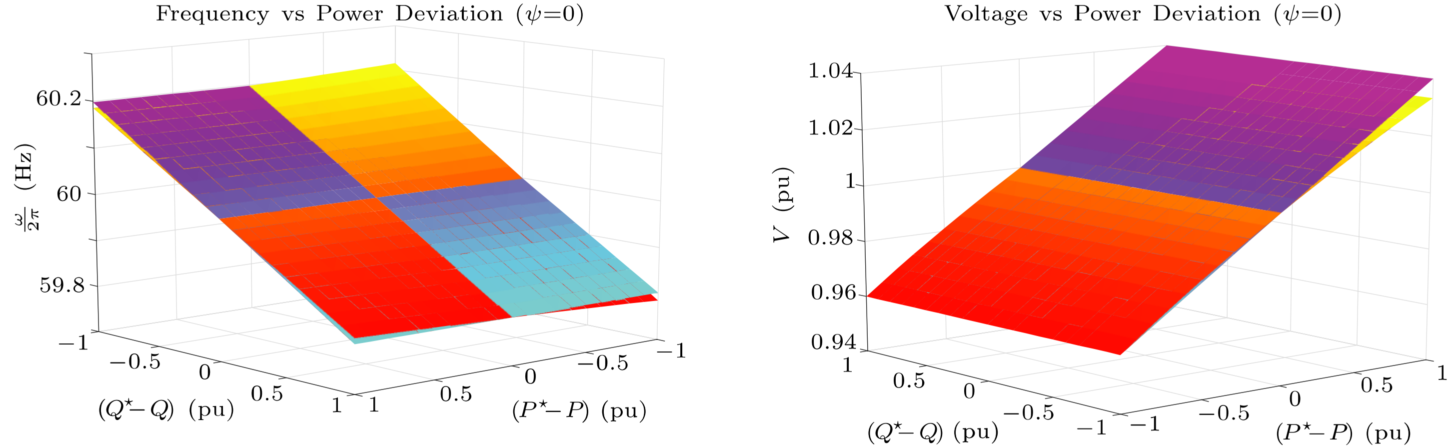
inductive lines

$\psi = \frac{\pi}{2}$



resistive lines

$\psi = 0$



[1] Johnson, et. al, “A generic primary-control model for grid-forming inverters: Towards interoperable operation & control,” HICSS, 2021.

[2] Ajala, Baeckeland, Dhople, Dominguez-Garcia, “Uncovering the Kuramoto model from full-order models of grid-forming inverter-based power networks,” CDC, 2021.

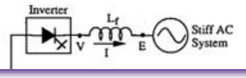
Unifying 25+ Years of GFM Technologies

1993

Control of Parallel Connected Inverters in Standalone ac Supply Systems

Mukul C. Chandorkar, *Student Member, IEEE*, Deepakraj M. Divan, *Member, IEEE*, and Rambabu Adapa, *Senior Member, IEEE*

***Abstract*—A scheme for controlling parallel-connected inverters in a standalone ac supply system is presented in this paper. This scheme is suitable for control of inverters in distributed source environments such as in isolated ac systems, large and**



2014

Synchronization of Parallel Single-Phase Inverters With Virtual Oscillator Control

Brian B. Johnson, *Member, IEEE*, Sairaj V. Dhople, *Member, IEEE*, Abdullah O. Hamadeh, and Philip T. Krein, *Fellow, IEEE*

***Abstract*—A method to synchronize and control a system of parallel single-phase inverters without communication is presented. Inspired by the phenomenon of synchronization in networks of coupled oscillators, we propose that each inverter be controlled**

proportion to their ratings. Focused on these challenges, this paper presents a method to synchronize and control a system of parallel single-phase inverters without communication. It is

2007

Virtual Synchronous Machine

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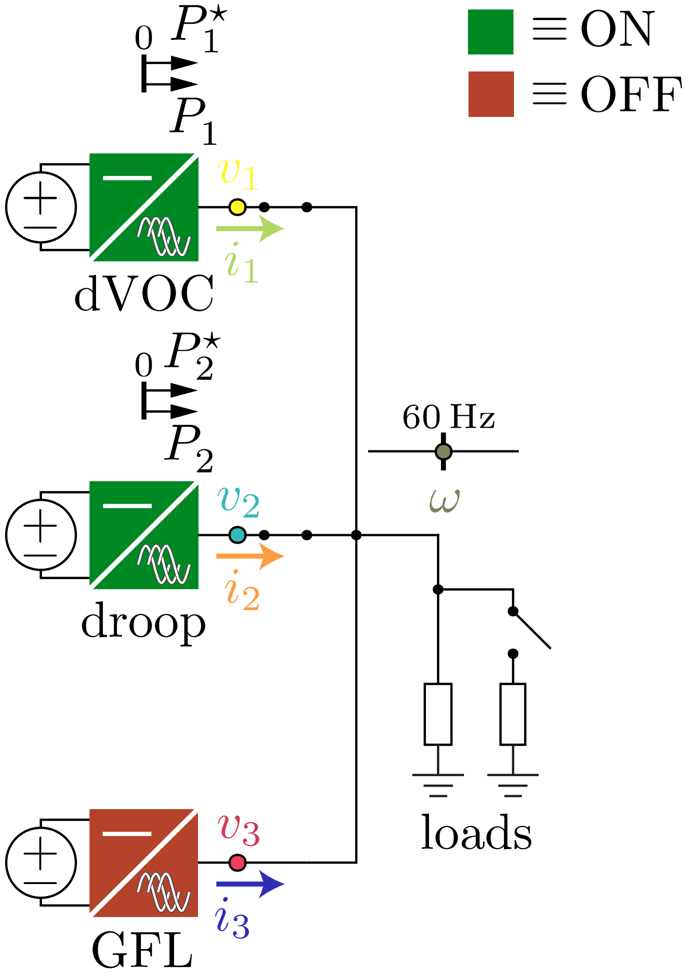
Demands in the area of electrical energy generation and distribution, as a result of energy policies, are leading to far reaching changes in the structure of the energy supply, which is

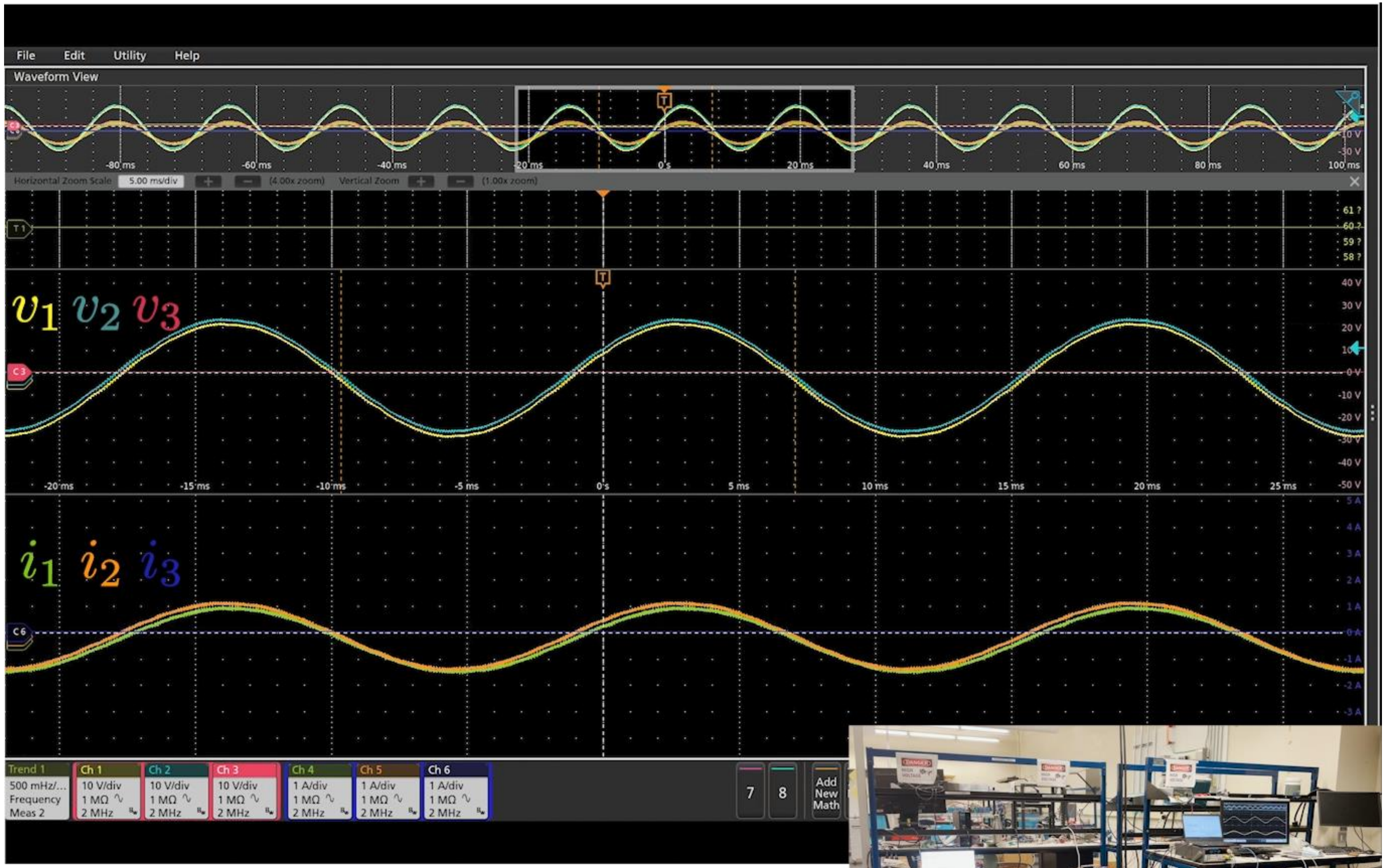
cause or promote the appearance and mitigation of various disturbances.

$$
\begin{aligned}
\tau_{\mathrm{f}} \frac{\mathrm{d}\omega}{\mathrm{d}t} &= -\omega + \omega_0 + \kappa_{\mathrm{d}}\left(\omega_{\mathrm{g}} - \omega\right) \\
&\quad + \kappa_{\mathrm{f}} \begin{bmatrix} 1 & 0 \end{bmatrix} \mathrm{R}(\psi - \tfrac{\pi}{2}) \begin{bmatrix} P^\star - P_{\mathrm{m}} \\ Q^\star - Q_{\mathrm{m}} \end{bmatrix}, \\
\tau_{\mathrm{v}} \frac{\mathrm{d}V}{\mathrm{d}t} &= f_{\mathrm{v}}(V) + \kappa_{\mathrm{v}} \begin{bmatrix} 0 & 1 \end{bmatrix} \mathrm{R}(\psi - \tfrac{\pi}{2}) \begin{bmatrix} p^\star - p_{\mathrm{m}} \\ q^\star - q_{\mathrm{m}} \end{bmatrix}, \\
\frac{1}{\omega_0} \frac{\mathrm{d}\eta}{\mathrm{d}t} &= \begin{bmatrix} 0 & 1 \end{bmatrix} \mathrm{R}(\alpha)\mathrm{R}(\delta)\mathrm{T}(\omega_0 t) v, \\
\frac{1}{\omega_0} \frac{\mathrm{d}\alpha}{\mathrm{d}t} &= \frac{k_{\mathrm{P}}}{\omega_0} \dot{\eta} + k_{\mathrm{I}} \eta, \\
\tau_{\mathrm{p}} \begin{bmatrix} \dot{P}_{\mathrm{m}} \\ \dot{Q}_{\mathrm{m}} \end{bmatrix} &= -\begin{bmatrix} P_{\mathrm{m}} \\ Q_{\mathrm{m}} \end{bmatrix} + \begin{bmatrix} P \\ Q \end{bmatrix},
\end{aligned}
$$

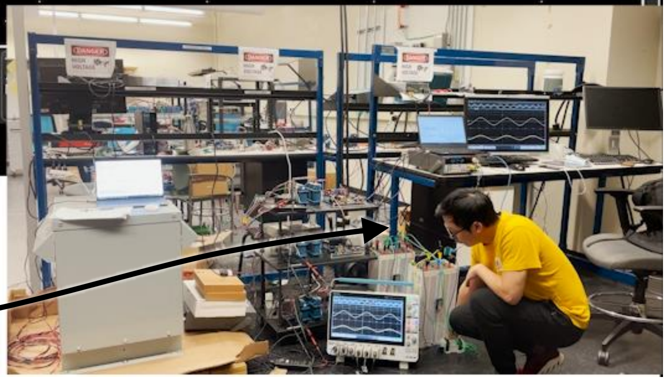
A unified model

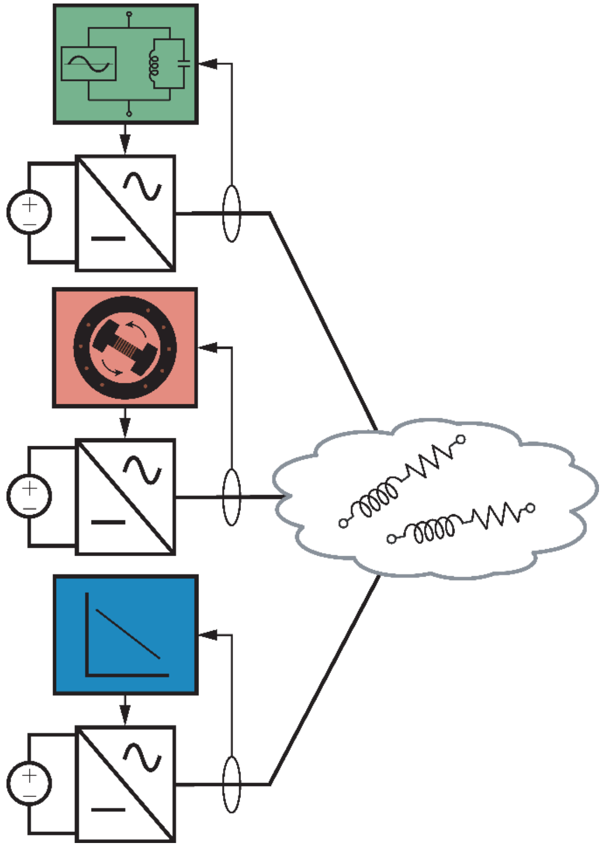
Showing Interoperability with a Mix of Control Types



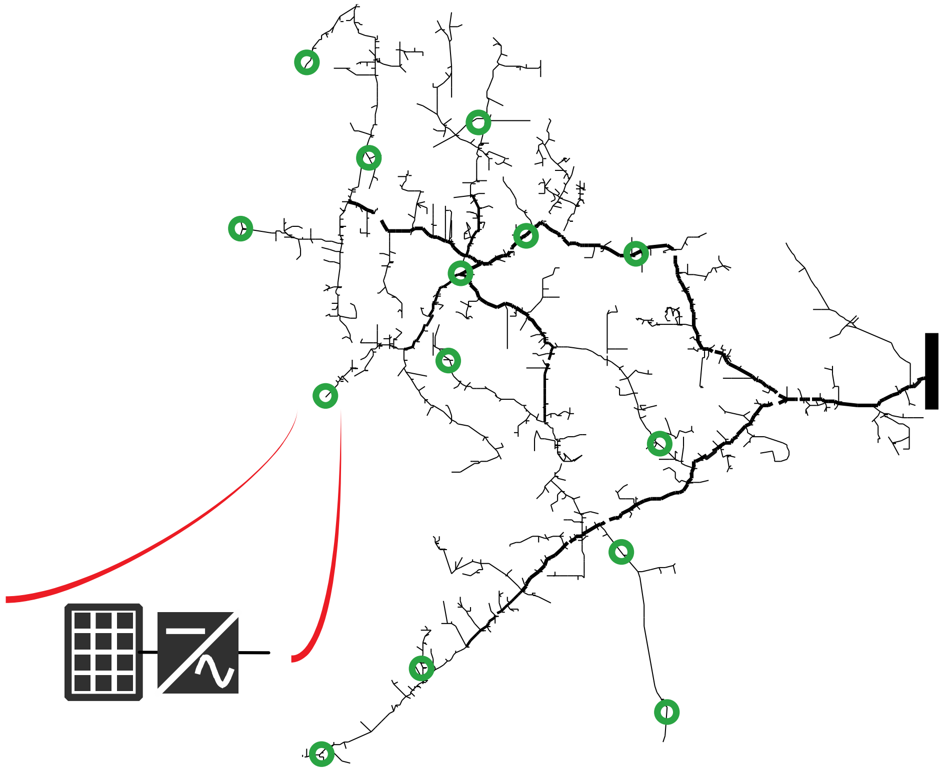


Weiqian Cai





Interoperability



Self-organizing GFM Networks

A Single-phase GFM Commercial Product

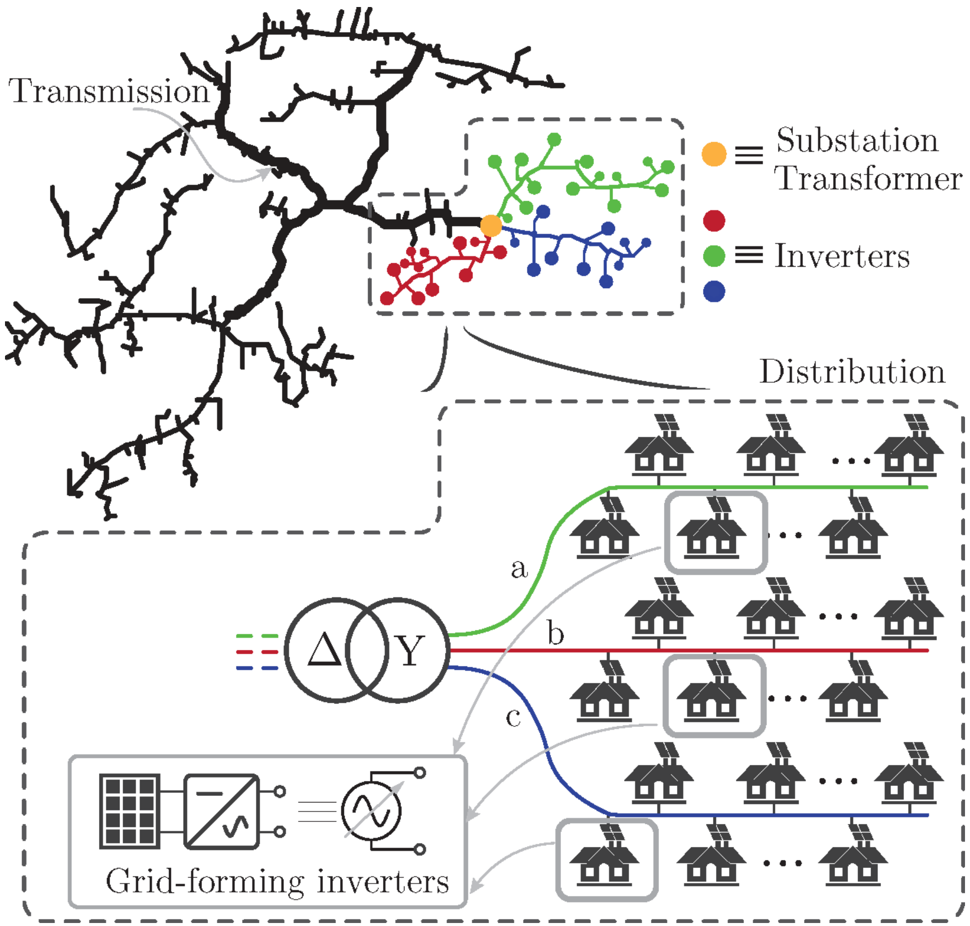
Features of the inverter building block

- 300 VA single-phase with droop-based GFM controls
- Bidirectional converter can interface PV or batteries

Over 39M+ Enphase inverters shipped for 12 GW of capacity as of September 2021

Building Three-phase Systems with Single-phase GFM Units

Can swarms of decentralized single-phase GFMs self-organize into a three-phase system?

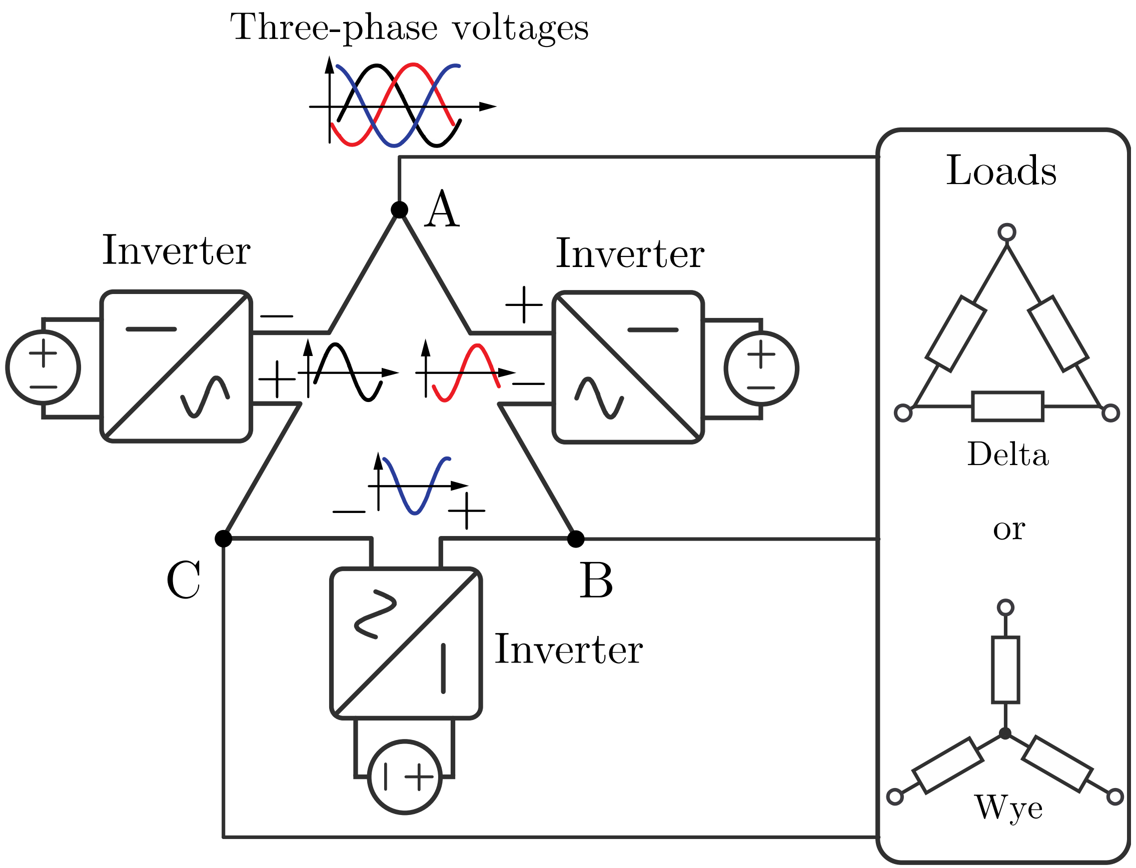


Could this system maintain phase balancing when islanded?

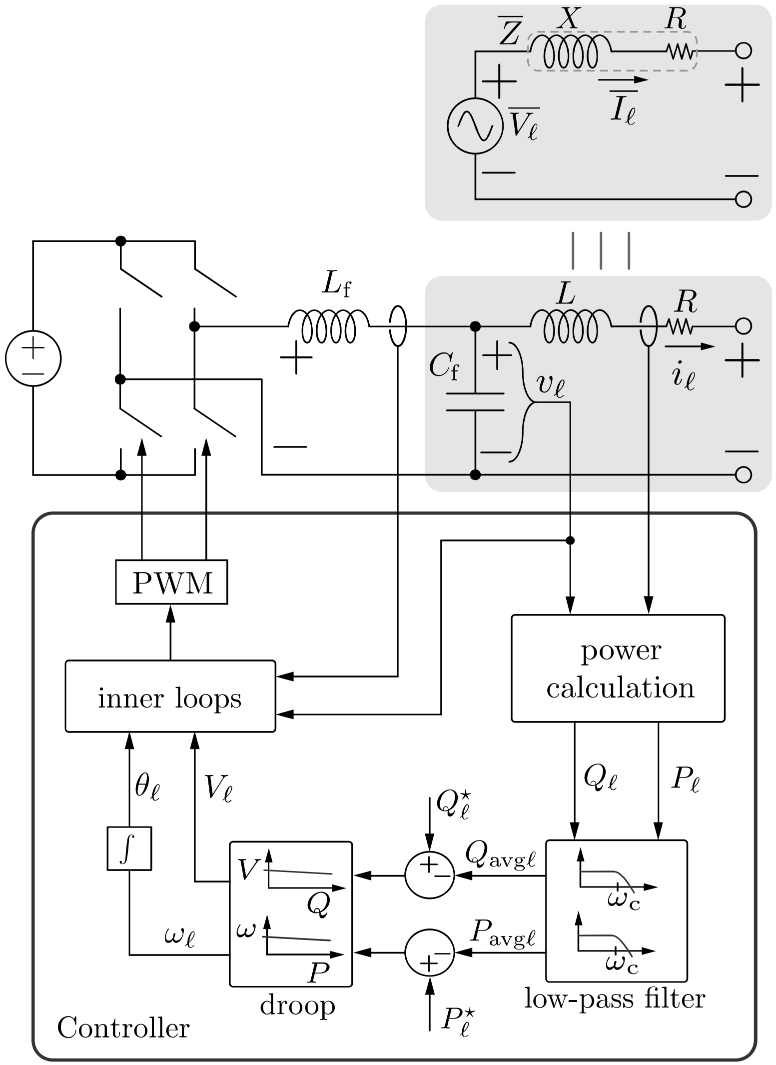
From Industry Challenges to Foundational Research

Industry partners have already prioritized three-phase system operation with single-phase units

Donny Zimmanck



Single-phase Inverter Model



A single-phase droop-controlled inverter

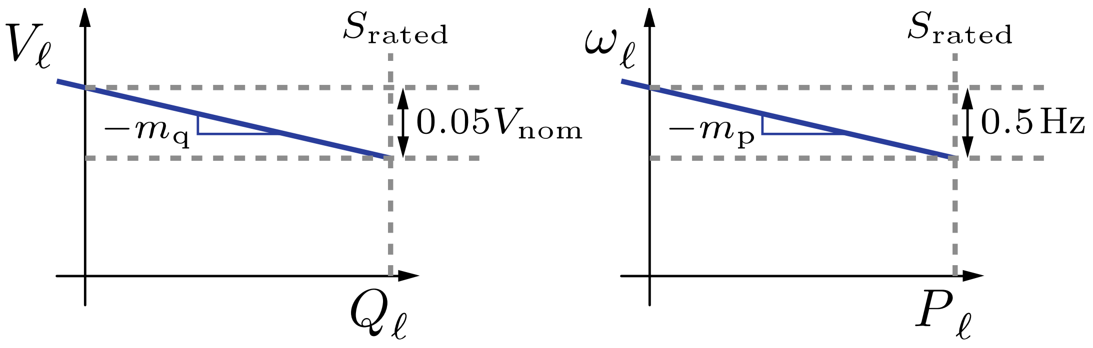
The $\ell \in 1, 2, 3$ inverter has the droop laws

$$V_\ell = V_\mathrm{nom} - m_\mathrm{q} \left(Q_{\mathrm{avg},\ell} - Q_\ell^\star \right),$$
$$\omega_\ell = \omega_\mathrm{nom} - m_\mathrm{p} \left(P_{\mathrm{avg},\ell} - P_\ell^\star \right),$$

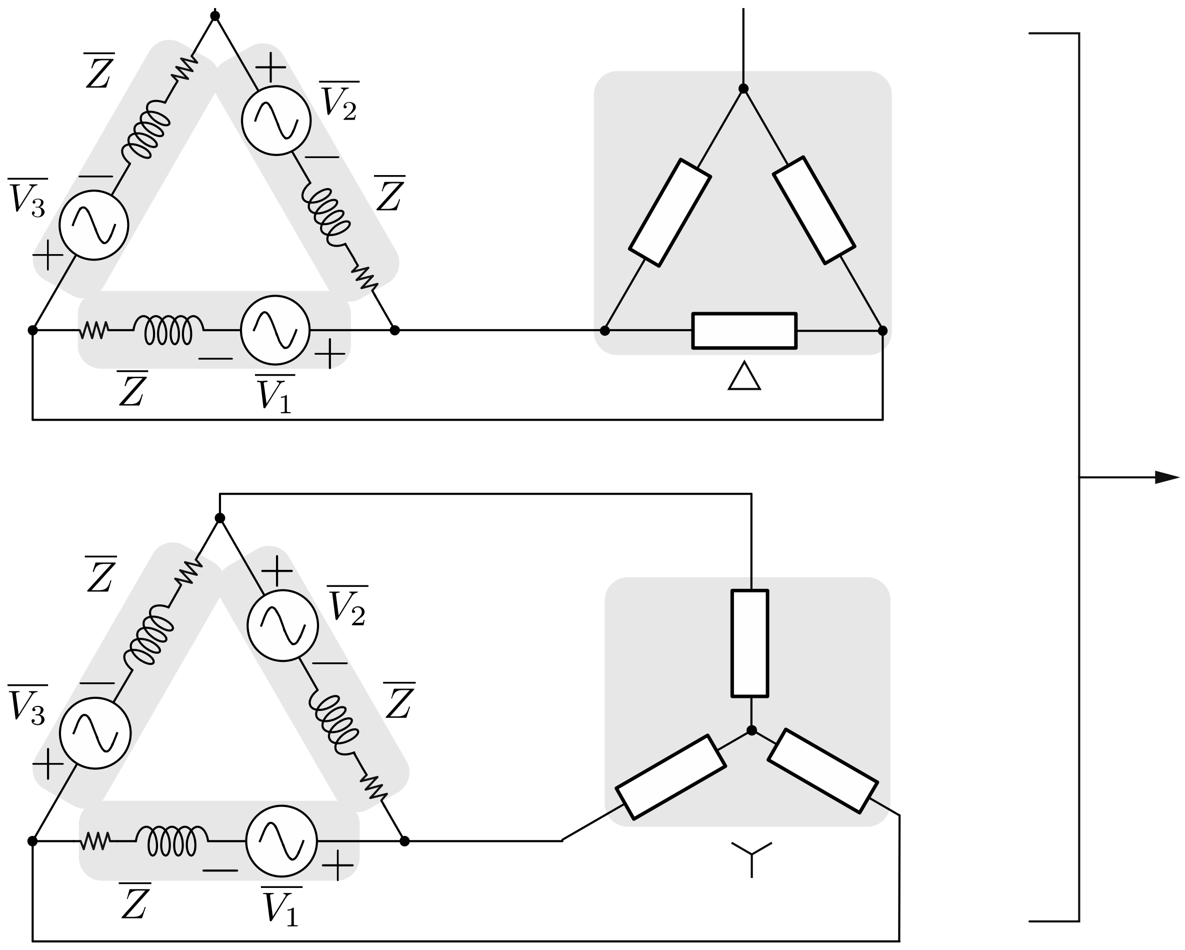
where

$$\dot{P}_{\mathrm{avg},\ell} = \omega_\mathrm{c} \left(P_\ell - P_{\mathrm{avg},\ell} \right),$$
$$\dot{Q}_{\mathrm{avg},\ell} = \omega_\mathrm{c} \left(Q_\ell - Q_{\mathrm{avg},\ell} \right),$$

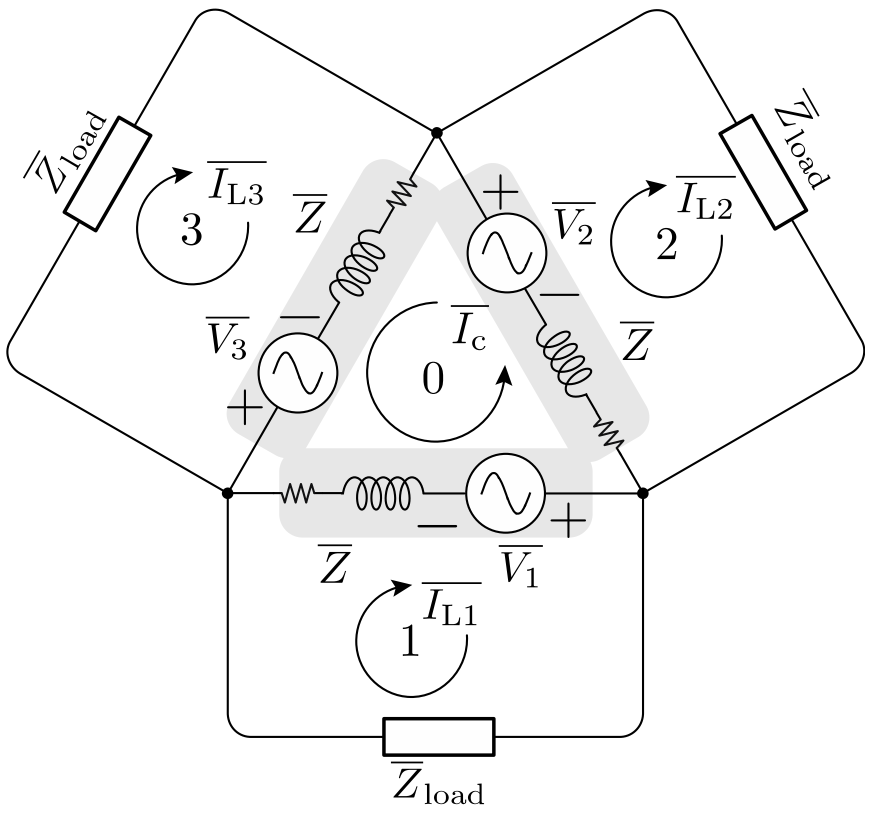
and slopes $m_\mathrm{q} = 0.05 \frac{V_\mathrm{nom}}{S_\mathrm{rated}}$, $m_\mathrm{p} = \frac{2\pi \times 0.5}{S_\mathrm{rated}}$. This gives the following in steady-state:



A Common Framework For Various Load Configurations



inverters with $\triangle$ & $\curlyvee$ loads



circuit model with mesh loops

A System Circuit Model

Circuit laws + control dynamics give

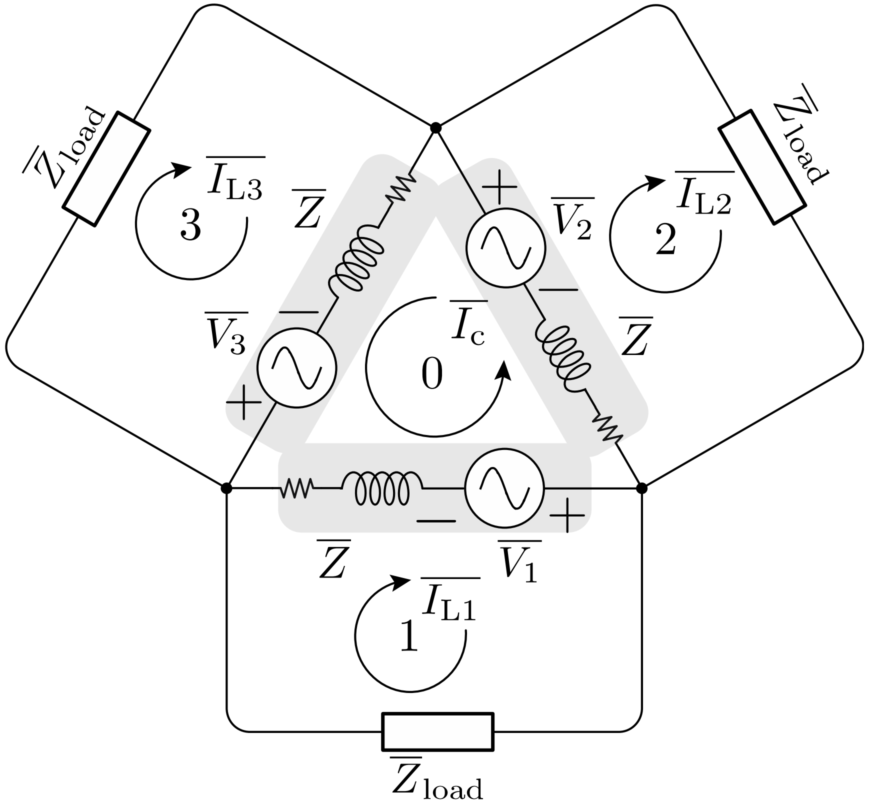
$$V_\ell \;=\; V_{\mathrm{nom}} - m_{\mathrm{q}} \sum_{k=1}^{3} \frac{V_k V_\ell}{|\overline{Z}_{\mathrm{loop}}|} \sin\left(\theta_{k\ell} - \phi\right),$$

$$\dot{\theta}_\ell \;=\; \omega_{\mathrm{nom}} - m_{\mathrm{p}} \sum_{k=1}^{3} \frac{V_k V_\ell}{|\overline{Z}_{\mathrm{loop}}|} \cos\left(\theta_{k\ell} - \phi\right).$$

where $\overline{Z}_{\mathrm{loop}} = 3\overline{Z}$ and $\phi = \tan^{-1}\left(X/R\right)$. Next, focus on angle difference dynamics

$$\dot{\theta}_{21} = \dot{\theta}_2 - \dot{\theta}_1,$$

$$\dot{\theta}_{31} = \dot{\theta}_3 - \dot{\theta}_1.$$



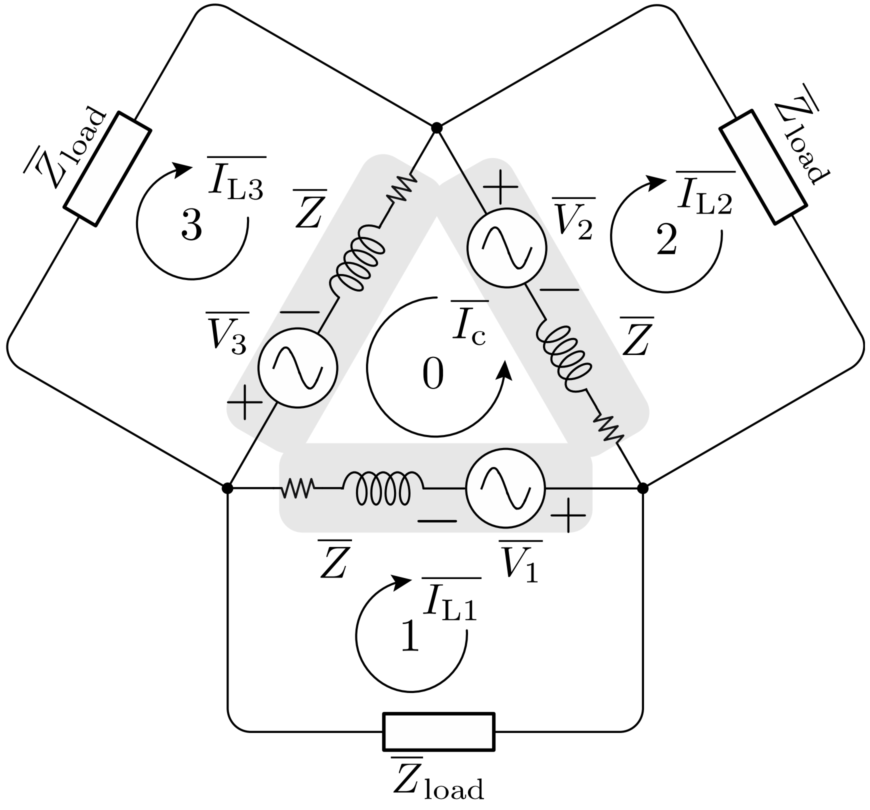
circuit model with mesh loops

Nonlinear Angular Dynamics

Algebra gives us

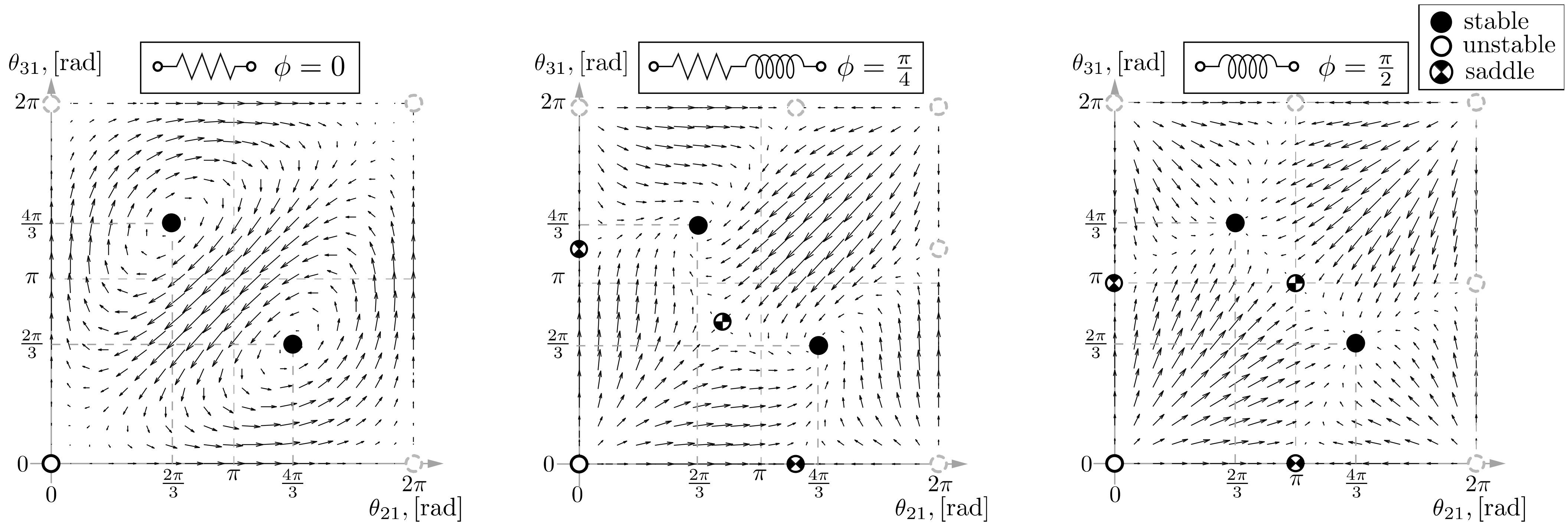
$$\dot{\theta}_{21} = \frac{m_{\mathrm{p}} V_2 V_1}{|\overline{Z}_{\mathrm{loop}}|}\cos\left(\theta_{21}-\phi\right) + \frac{m_{\mathrm{p}} V_3 V_1}{|\overline{Z}_{\mathrm{loop}}|}\cos\left(\theta_{31}-\phi\right) - \frac{m_{\mathrm{p}} V_2 V_3}{|\overline{Z}_{\mathrm{loop}}|}\cos\left(\theta_{32}-\phi\right) - \frac{m_{\mathrm{p}} V_2 V_1}{|\overline{Z}_{\mathrm{loop}}|}\cos\left(\theta_{12}-\phi\right),$$

$$\dot{\theta}_{31} = \frac{m_{\mathrm{p}} V_3 V_1}{|\overline{Z}_{\mathrm{loop}}|}\cos\left(\theta_{31}-\phi\right) + \frac{m_{\mathrm{p}} V_2 V_1}{|\overline{Z}_{\mathrm{loop}}|}\cos\left(\theta_{21}-\phi\right) + \frac{m_{\mathrm{p}} V_2 V_3}{|\overline{Z}_{\mathrm{loop}}|}\cos\left(\theta_{23}-\phi\right) + \frac{m_{\mathrm{p}} V_1 V_3}{|\overline{Z}_{\mathrm{loop}}|}\cos\left(\theta_{13}-\phi\right).$$



circuit model with mesh loops

Angle Difference Dynamics



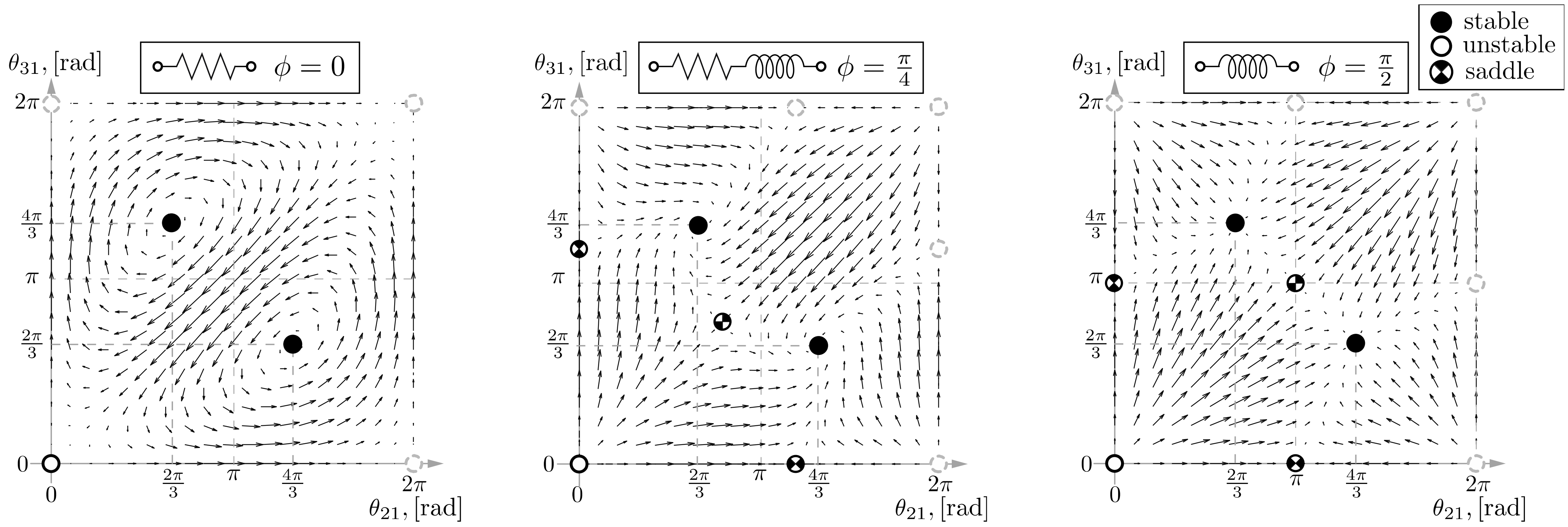
Substitute voltage droop laws, ignore $m_p^2, m_q^2, m_p m_q$ terms, and let $K = m_p V_{nom}^2 / |\overline{Z}_{loop}|$ to get:

$$\dot{\theta}_{21} \approx K(2\sin\theta_{21} + \sin\theta_{31} + \sin(\theta_{21} - \theta_{31}))\sin\phi + K(\cos\theta_{31} - \cos(\theta_{21} - \theta_{31}))\cos\phi,$$

$$\dot{\theta}_{31} \approx K(2\sin\theta_{31} + \sin\theta_{21} + \sin(\theta_{31} - \theta_{21}))\sin\phi + K(\cos\theta_{21} - \cos(\theta_{31} - \theta_{21}))\cos\phi.$$

[1] Lu, Dhople, Zimmanck, Johnson, "Spontaneous phase balancing in delta-connected single-phase grid-forming inverters," TPEL, 2021.

Angle Difference Dynamics



The equilibria are below, where $\sigma_1 = 2\pi + 2\tan^{-1}(-3\tan\phi)$, $\sigma_2 = 2\tan^{-1}(3\tan\phi)$.

$$(\theta_{21,\mathrm{eq}},\ \theta_{31,\mathrm{eq}})^{\bullet} = \left\{\left(\tfrac{2\pi}{3},\ \tfrac{4\pi}{3}\right), \left(\tfrac{4\pi}{3},\ \tfrac{2\pi}{3}\right)\right\},$$

$$(\theta_{21,\mathrm{eq}},\ \theta_{31,\mathrm{eq}})^{\circ} = \{(0,0)\},$$

$$(\theta_{21,\mathrm{eq}},\ \theta_{31,\mathrm{eq}})^{\otimes} = \{(0,\sigma_1), (\sigma_1, 0),\ (\sigma_2, \sigma_2)\},$$

[1] Lu, Dhople, Zimmanck, Johnson, "Spontaneous phase balancing in delta-connected single-phase grid-forming inverters," TPEL, 2021.

Small-signal Modeling

The linearized system is

$$\begin{bmatrix} \Delta\dot{\theta}_{21} \\ \Delta\dot{\theta}_{31} \end{bmatrix} = \mathcal{J}(\theta_{21,\mathrm{eq}}, \theta_{31,\mathrm{eq}}) \begin{bmatrix} \Delta\theta_{21} \\ \Delta\theta_{31} \end{bmatrix},$$

where

$$\mathcal{J}(\theta_{21,\mathrm{eq}}, \theta_{31,\mathrm{eq}}) = K \left[\begin{array}{c|c} \begin{array}{c}(2\cos\theta_{21,\mathrm{eq}} + \cos(\theta_{21,\mathrm{eq}} - \theta_{31,\mathrm{eq}}))\sin\phi \\ +\sin(\theta_{21,\mathrm{eq}} - \theta_{31,\mathrm{eq}})\cos\phi\end{array} & \begin{array}{c}(\cos\theta_{31,\mathrm{eq}} - \cos(\theta_{21,\mathrm{eq}} - \theta_{31,\mathrm{eq}}))\sin\phi \\ +(\sin(\theta_{31,\mathrm{eq}} - \theta_{21,\mathrm{eq}}) - \sin\theta_{31,\mathrm{eq}})\cos\phi\end{array} \\ \hline \begin{array}{c}(\cos\theta_{21,\mathrm{eq}} - \cos(\theta_{31,\mathrm{eq}} - \theta_{21,\mathrm{eq}}))\sin\phi \\ +(\sin(\theta_{21,\mathrm{eq}} - \theta_{31,\mathrm{eq}}) - \sin\theta_{21,\mathrm{eq}})\cos\phi\end{array} & \begin{array}{c}(2\cos\theta_{31,\mathrm{eq}} + \cos(\theta_{31,\mathrm{eq}} - \theta_{21,\mathrm{eq}}))\sin\phi \\ +\sin(\theta_{31,\mathrm{eq}} - \theta_{21,\mathrm{eq}})\cos\phi\end{array} \end{array}\right],$$

and the eigenvalues for all equilibria within $\phi \in (0, \pi/2]$ are

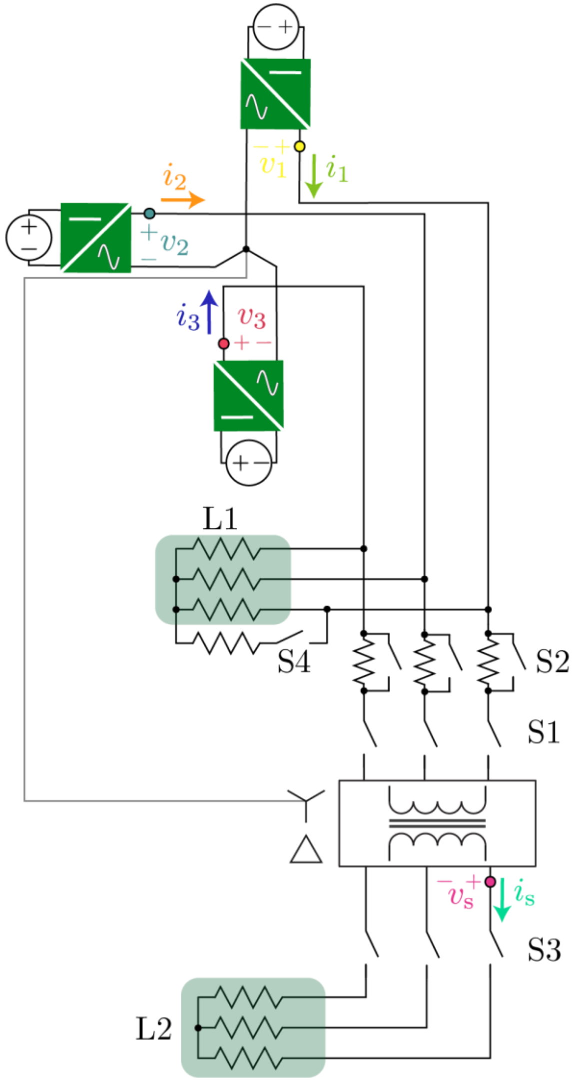
$$\lambda_{1,2}^{\bullet} = -\frac{3}{2}K(\sin\phi \pm \jmath\cos\phi) \longrightarrow \Re(\lambda_{1,2}^{\bullet}) < 0 \longrightarrow \text{stable,}$$

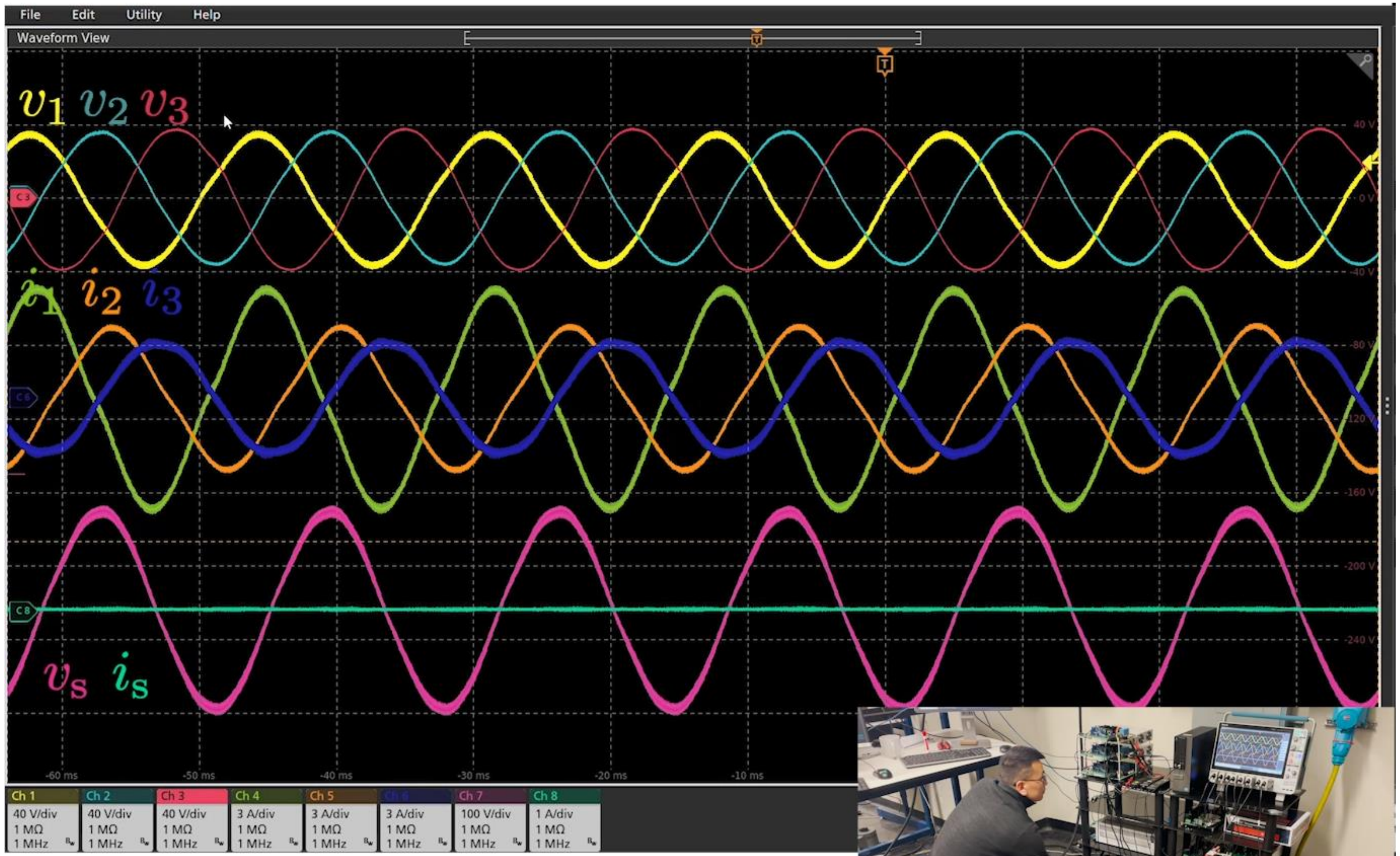
$$\lambda_{1,2}^{\circ} = 3K\sin\phi \longrightarrow \Re(\lambda_{1,2}^{\circ}) > 0 \longrightarrow \text{unstable,}$$

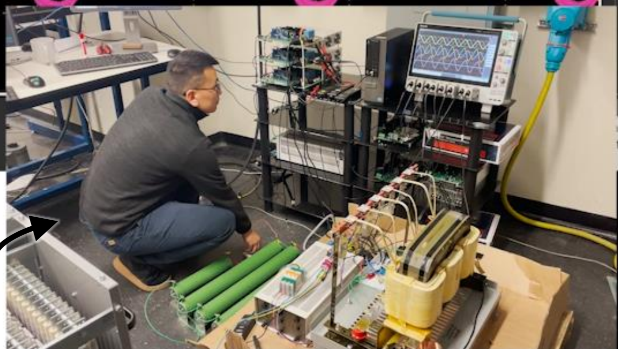
$$\lambda_{1}^{\oplus} = -3K\sin\phi, \quad \lambda_{2}^{\oplus} = 9K\frac{1+\tan^2\phi}{1+9\tan^2\phi}\sin\phi \longrightarrow \Re(\lambda_{1}^{\oplus}) < 0 \text{ and } \Re(\lambda_{2}^{\oplus}) > 0 \longrightarrow \text{saddle.}$$

[1] Lu, Dhople, Zimmanck, Johnson, "Spontaneous phase balancing in delta-connected single-phase grid-forming inverters," TPEL, 2021.

Self-balancing Single-phase GFM Hardware Results



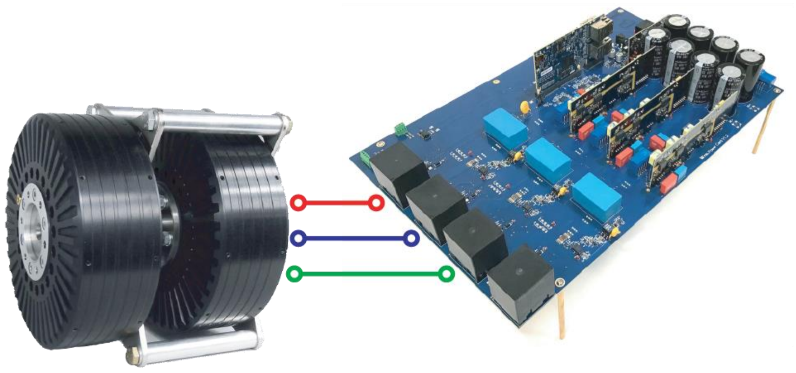




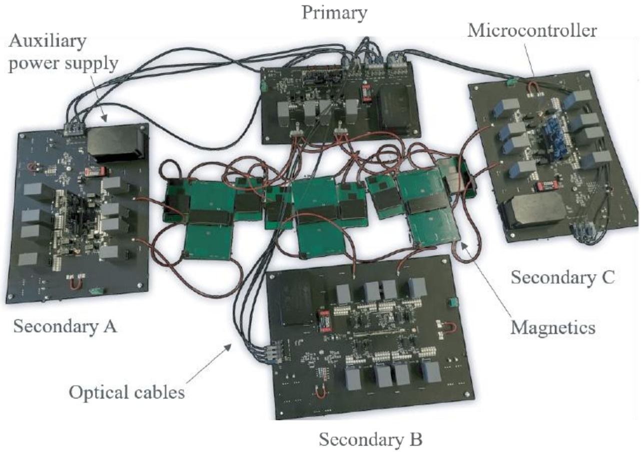
Dr. Minghui Lu

Thanks for Your Attention!

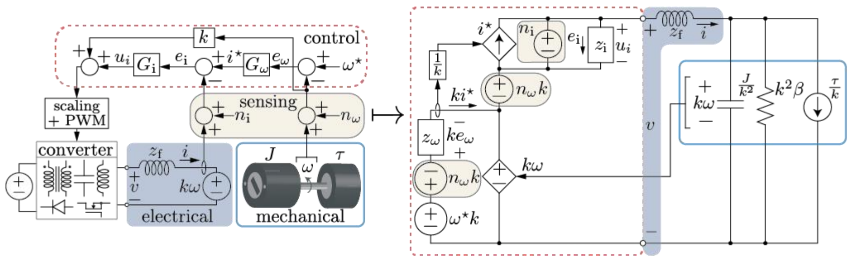
My other work building & analyzing future energy systems at all scales



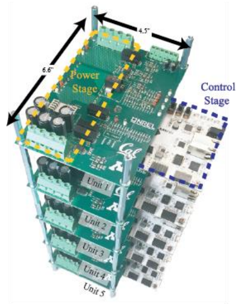
electromechanics



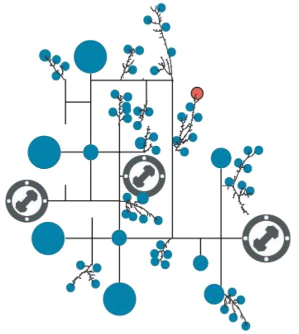
medium-voltage electronics



multiphysics modeling



dc-dc converters



system dynamics