# Periodic Material-Based Seismic Base Isolators for Small Modular Reactors

NEET-1 Annual Meeting
September 29, 2015


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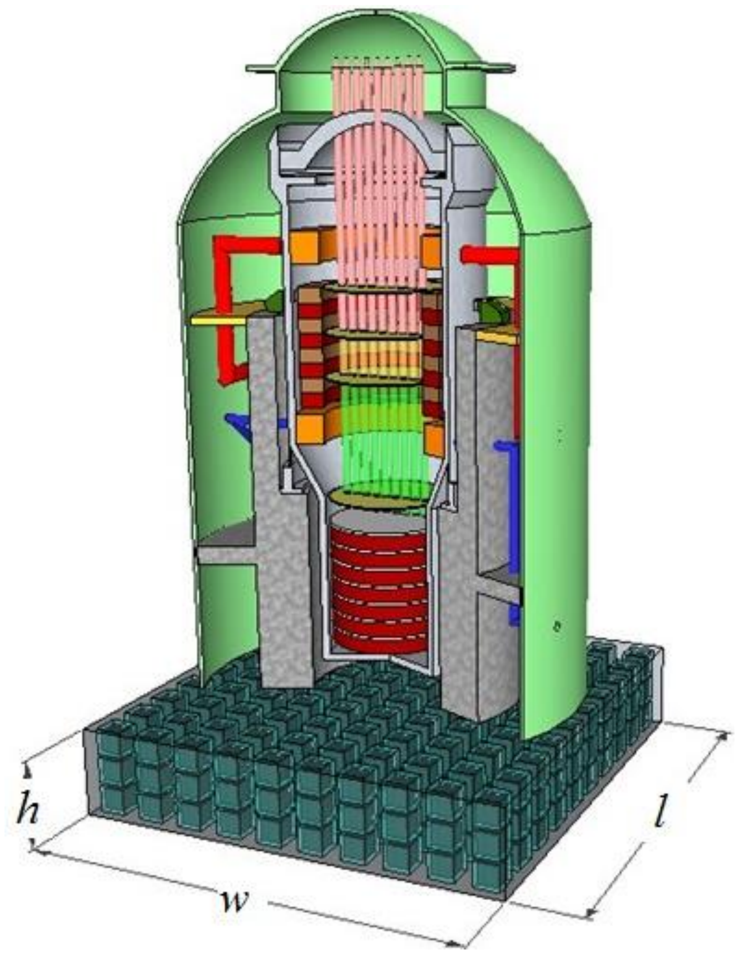
# Project overview

**Purpose:**

To develop a periodic foundation that can completely obstruct or change the energy pattern of the earthquake before it reaches the structure of small modular reactors (SMR).

**Scopes:**

(1) Perform comprehensive, analytical study on periodic foundations.
(2) Design a SMR model with periodic foundations.
(3) Verify the effectiveness of periodic foundations through shake table tests.
(4) Perform finite element simulation of SMR supported by periodic foundations.



# Project schedule

<table>
<tr><th rowspan="2">Task</th><th>2014</th><th colspan="4">2015</th><th colspan="4">2016</th><th colspan="3">2017</th></tr>
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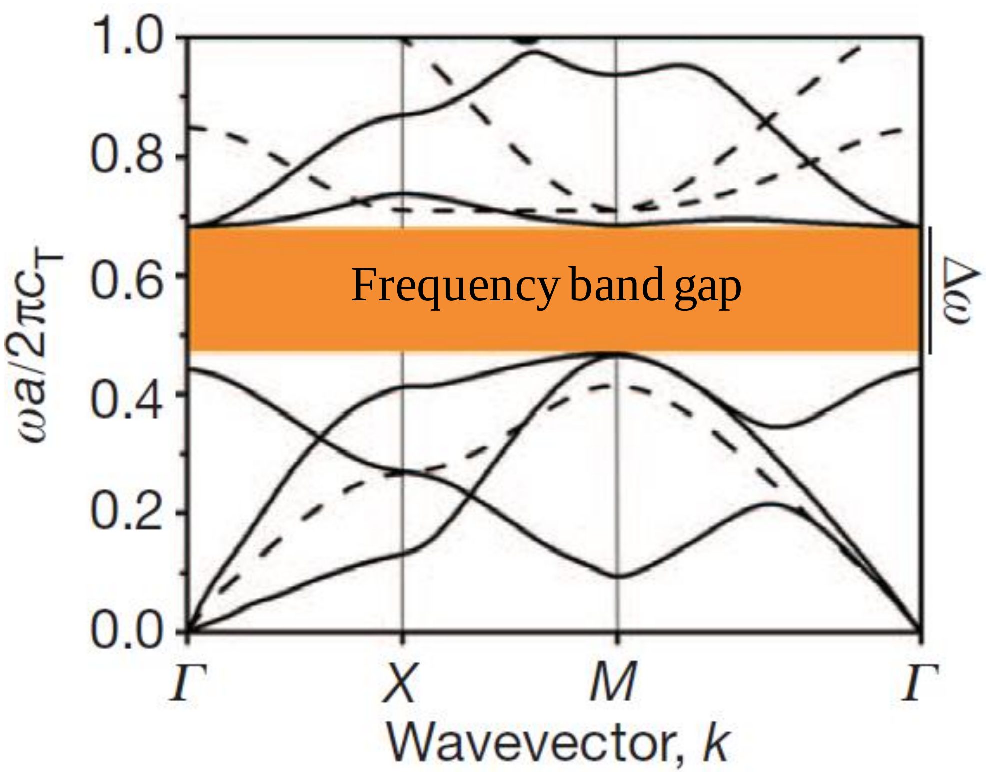
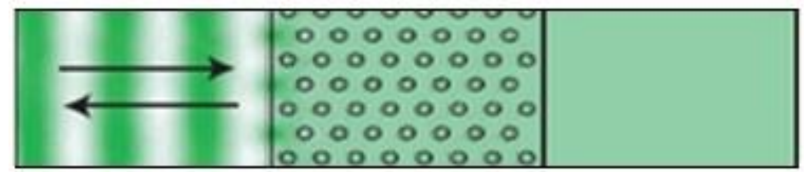
# Wave propagation in phononic crystal

Phononic crystal is a novel composite developed in solid-state-physics.

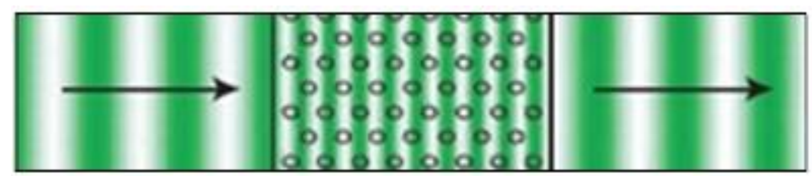
**Typical dispersion curve [1]**




**Wave Propagation [2]**



Wave propagation with frequency within the frequency band gap



Wave propagation with frequency outside of the frequency band gap


**[1] Maldovan, M. (2013). Sound and heat revolutions in phononics. *Nature, 503*(7475), 209-217.**

**[2] Thomas, E. L., Gorishnyy, T., & Maldovan, M. (2006). Phononics: Colloidal crystals go hypersonic. *Nature materials, 5*(10), 773-774.**

# Calculating dispersion curve

Governing equation of motion for a continuum body with isotropic elastic material

$$\rho(\mathbf{r})\frac{\partial^2 \mathbf{u}}{\partial t^2} = \nabla\left\{\left[\lambda(\mathbf{r}) + 2\mu(\mathbf{r})\right]\left(\nabla\cdot\mathbf{u}\right)\right\} - \nabla\times\left[\mu(\mathbf{r})\nabla\times\mathbf{u}\right] \qquad \text{Eq.1}$$

Where: $\mathbf{r}$ is coordinate vector
$\mathbf{u}(\mathbf{r})$ is displacement vector
$\rho(\mathbf{r})$ is the density
$\lambda(\mathbf{r})$ and $\mu(\mathbf{r})$ are the Lamé constant

Periodic boundary condition equation:

$$\mathbf{u}(\mathbf{r}+\mathbf{a},t) = e^{i\mathbf{K}\cdot\mathbf{a}}\mathbf{u}(\mathbf{r},t) \qquad \text{Eq.2}$$

Where: $\mathbf{K}$ is the wave vector
$\mathbf{a}$ is unit cell size

# Calculating dispersion curve

Applying the periodic boundary condition (Eq.2) to the governing equation, (Eq.1), the wave equation can be transferred into eigen value problem as follow:

$$\left(\mathbf{\Omega}(\mathbf{K}) - \omega^2\mathbf{M}\right).\mathbf{u} = 0 \qquad \text{Eq.3}$$

Where: $\mathbf{\Omega}$ is the stiffness matrix
$\mathbf{M}$ is the mass matrix

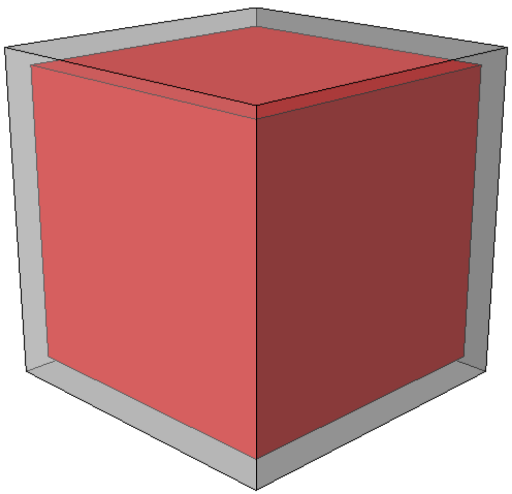
For each wave vector (**K**) a series of corresponding frequencies ($\omega$) can be obtained.

# Calculating dispersion curve

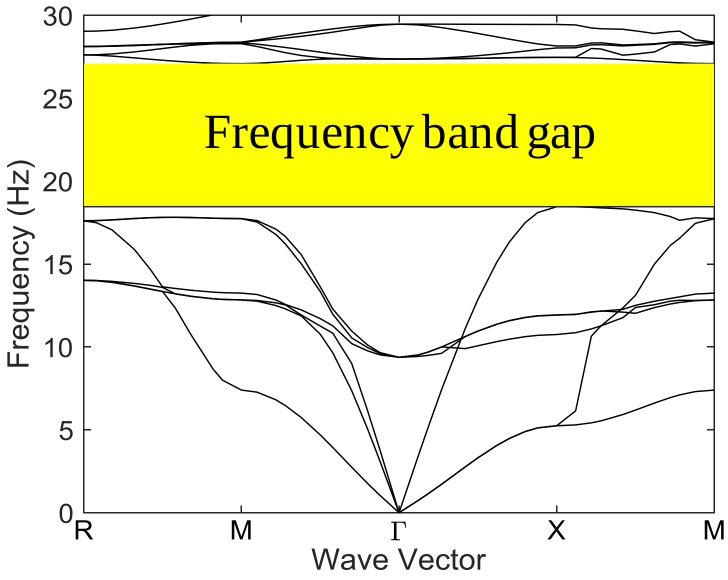
## Eigen value problem:

$$\left(\boldsymbol{\Omega}(\mathbf{K}) - \omega^2 \mathbf{M}\right).\mathbf{u} = 0$$

Infinite number of unit cells condition



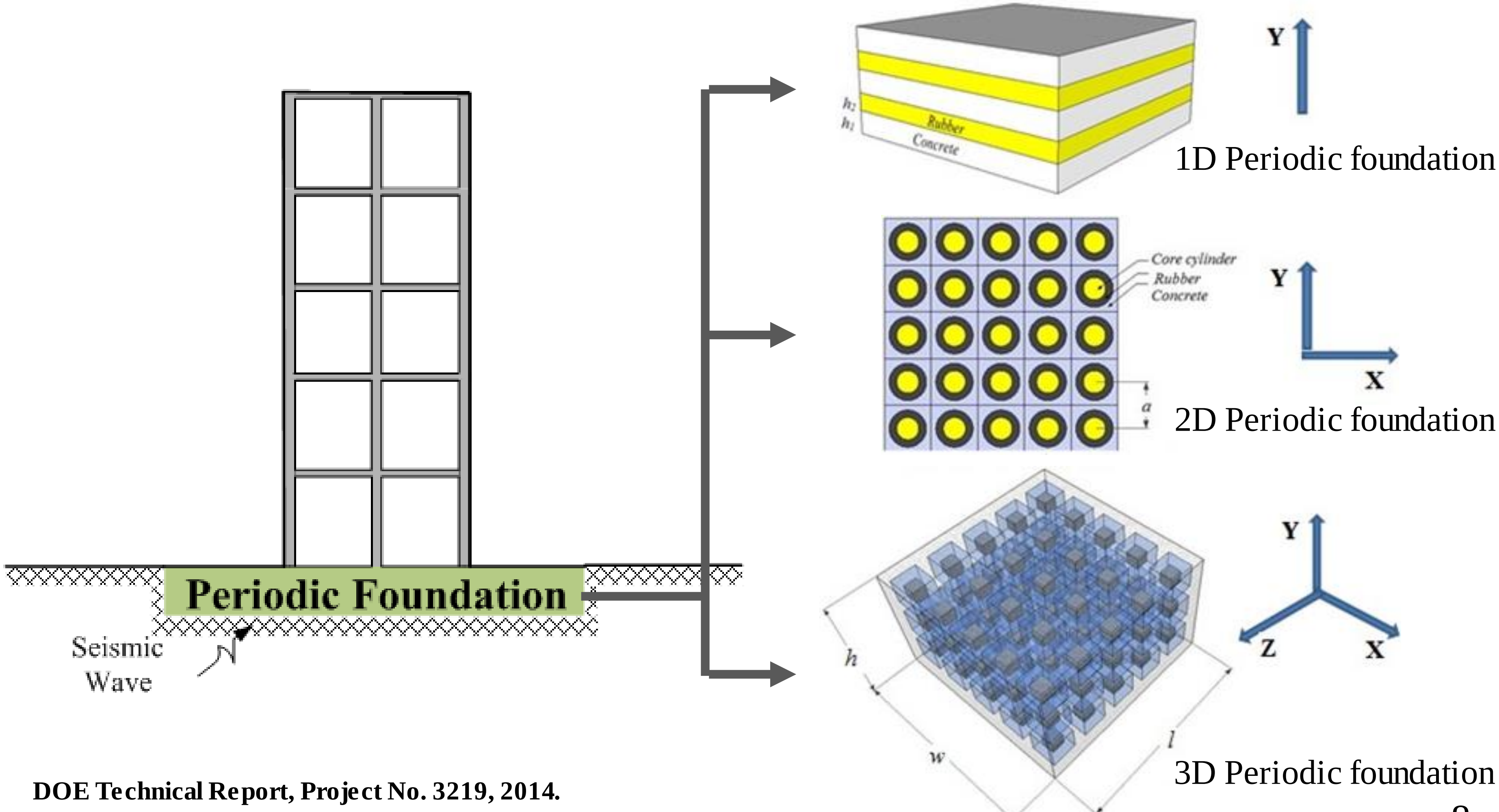
Typical two-component 3D periodic foundation




Typical dispersion curve

# Application of phononic crystal

In civil engineering field



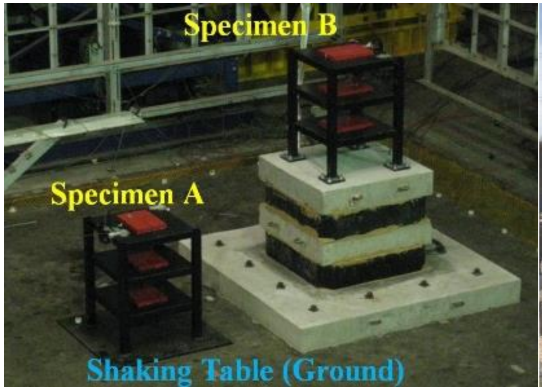

**DOE Technical Report, Project No. 3219, 2014.**

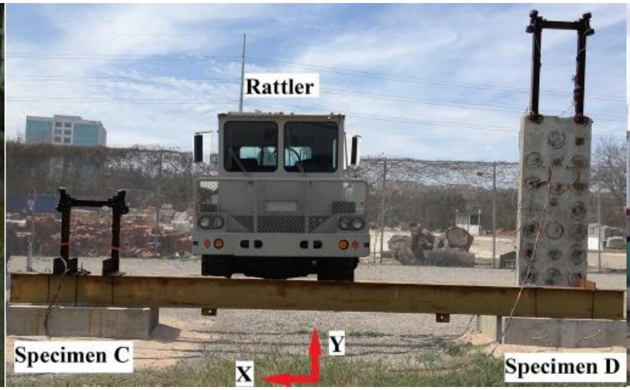
# Experimental study on periodic foundation

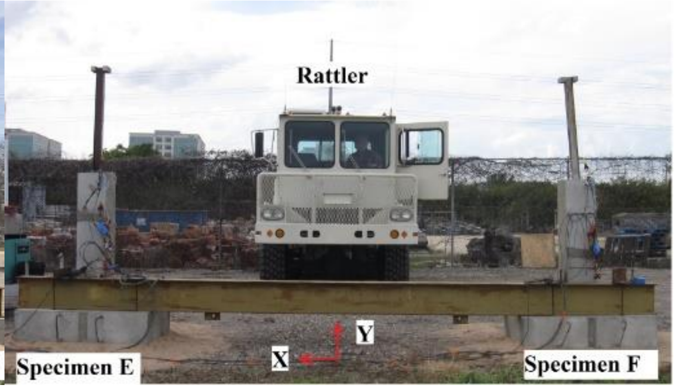
### 1D Periodic foundation

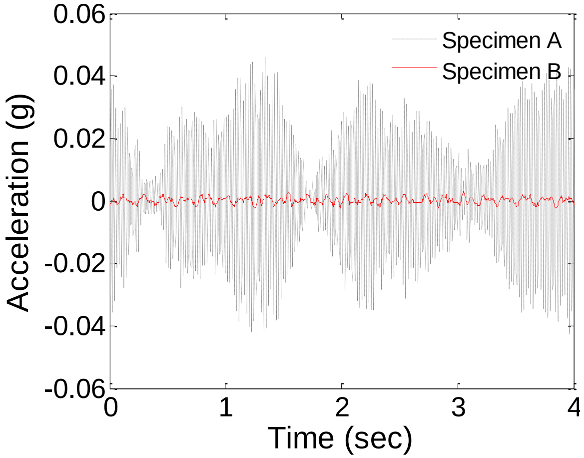
### 2D Periodic foundation

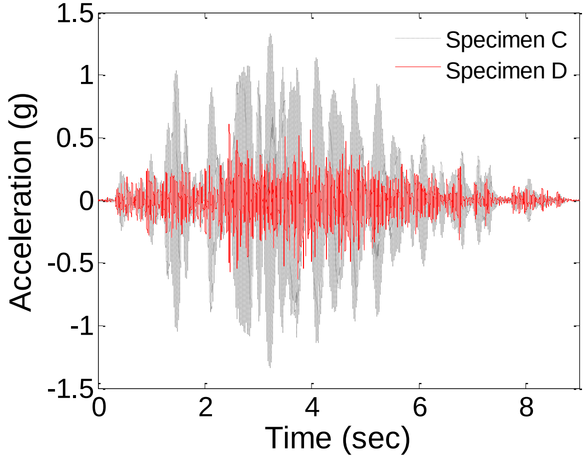
### 3D Periodic foundation

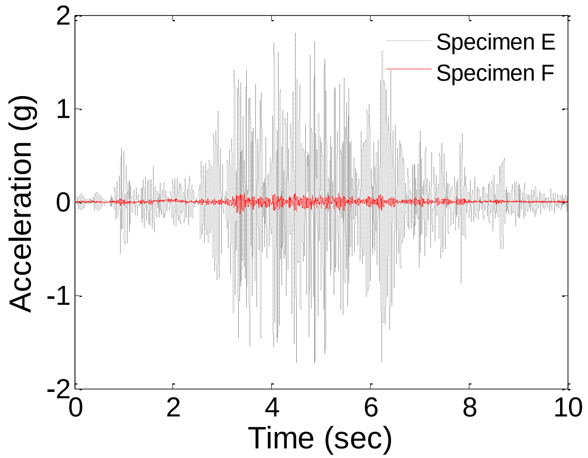


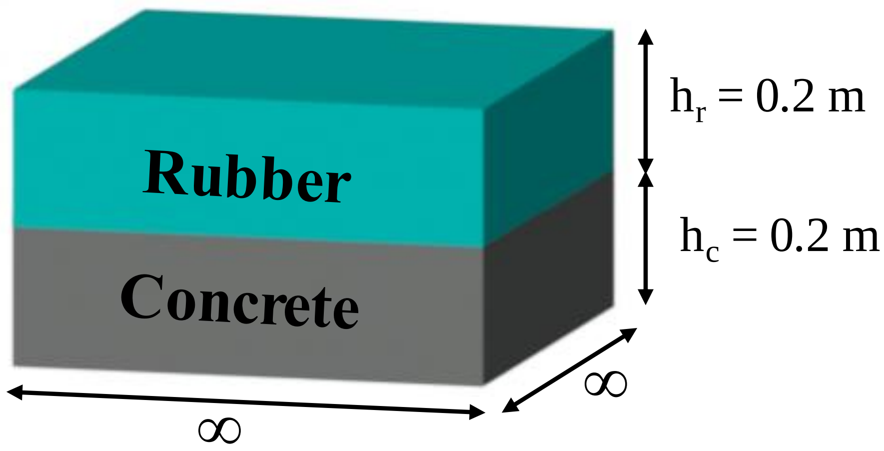




**DOE Technical Report, Project No. 3219, 2014.**

# 1D Periodic Foundations

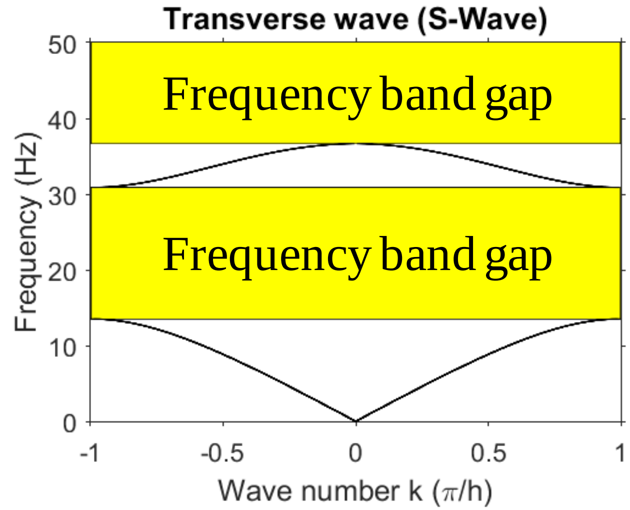
One unit cell of 1D periodic foundation

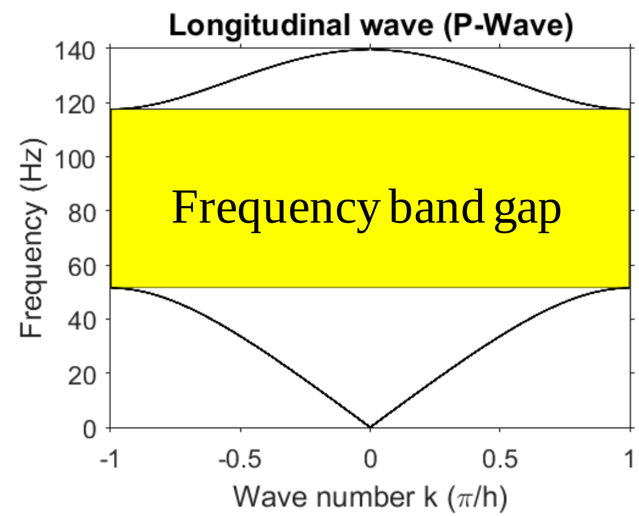



Fix material properties

| Material | Young's Modulus (Pa) | Density ($kg/m^3$) | Poisson's Ratio |
|---|---|---|---|
| Concrete | $3.14\times10^{10}$ | 2300 | 0.2 |
| Rubber | $5.8\times10^{5}$ | 1300 | 0.463 |

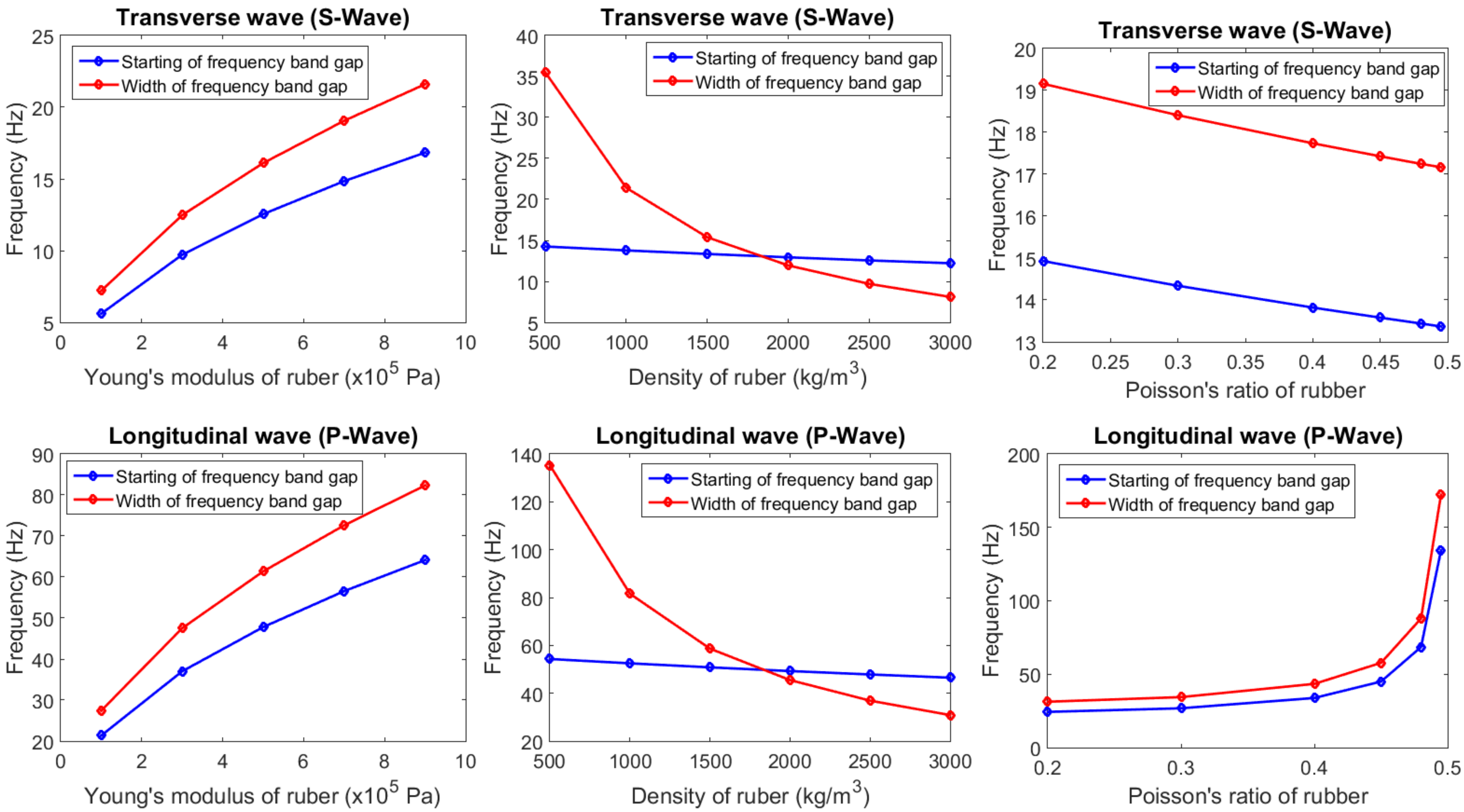
Dispersion curve for infinite number of unit cells



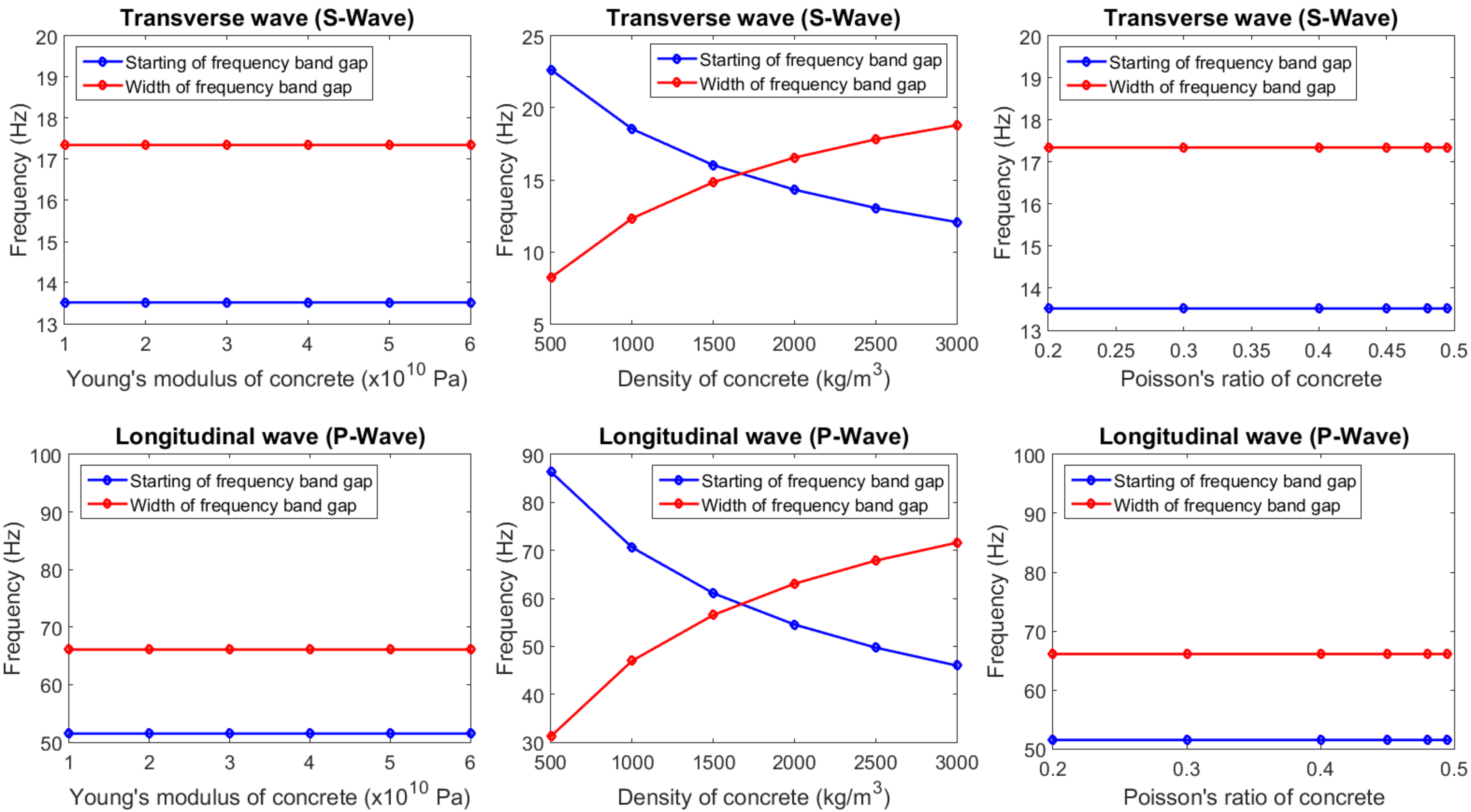
# Parametric study of 1D periodic foundations

## Effect of rubber material properties on the first frequency band gap



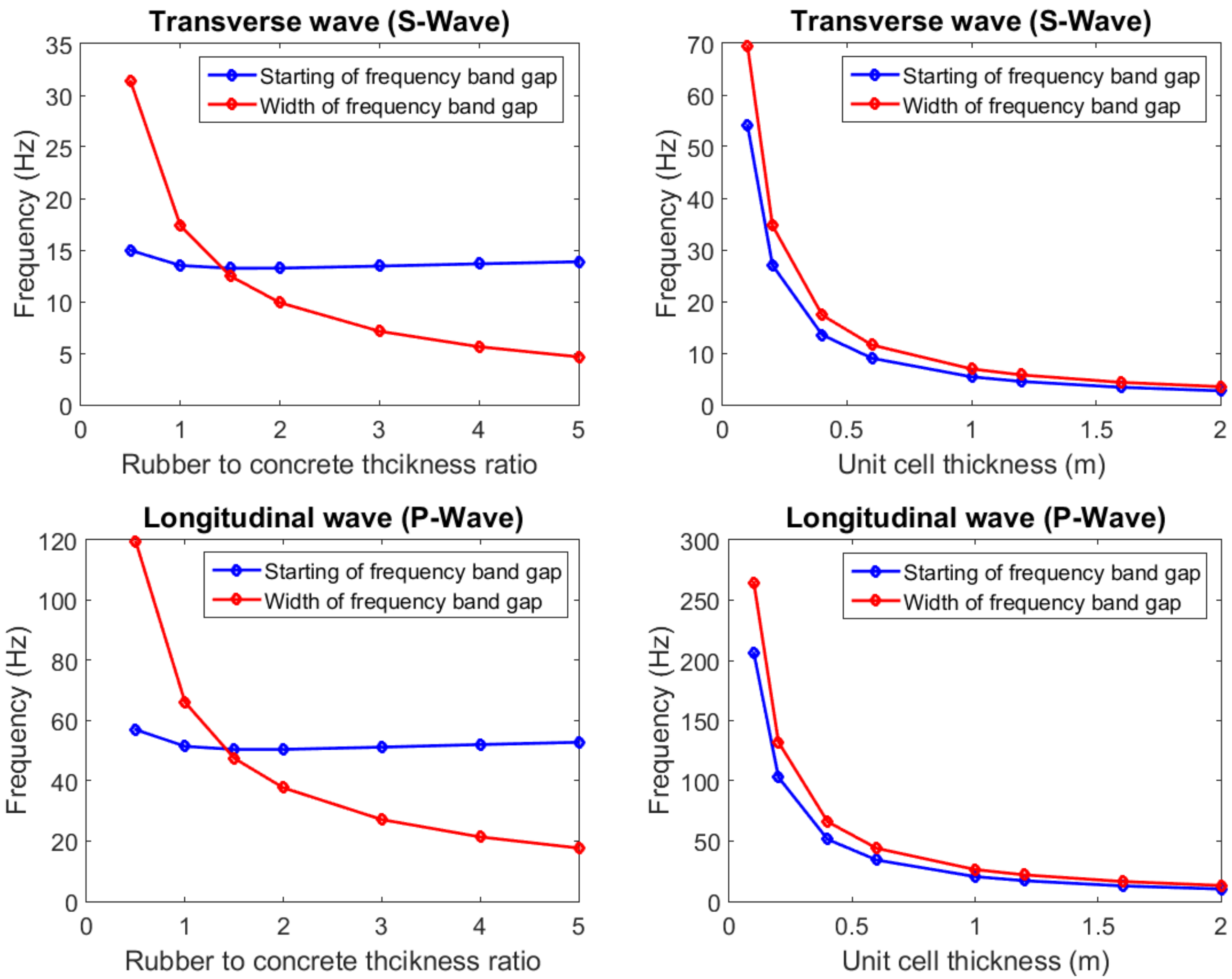
# Parametric study of 1D periodic foundations

## Effect of concrete material properties on the first frequency band gap



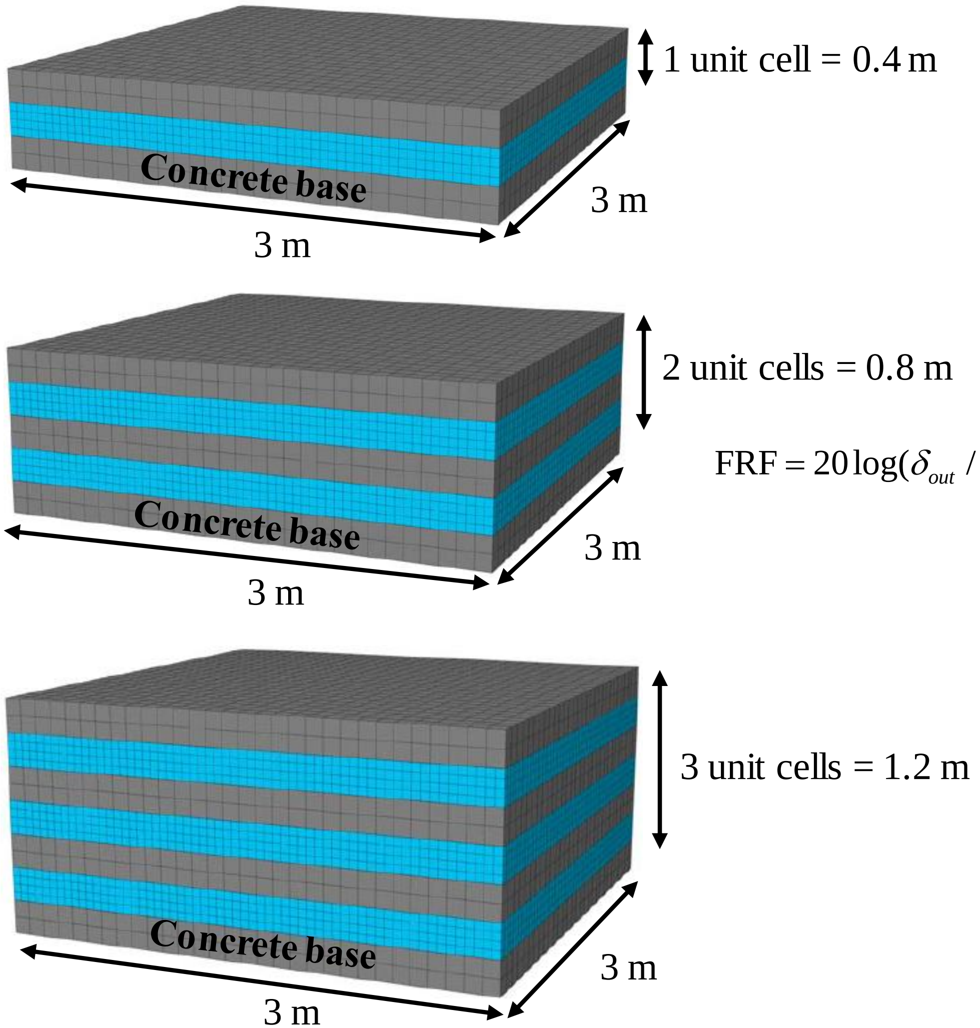
# Parametric study of 1D periodic foundations

## Effect of geometric properties on the first frequency band gap

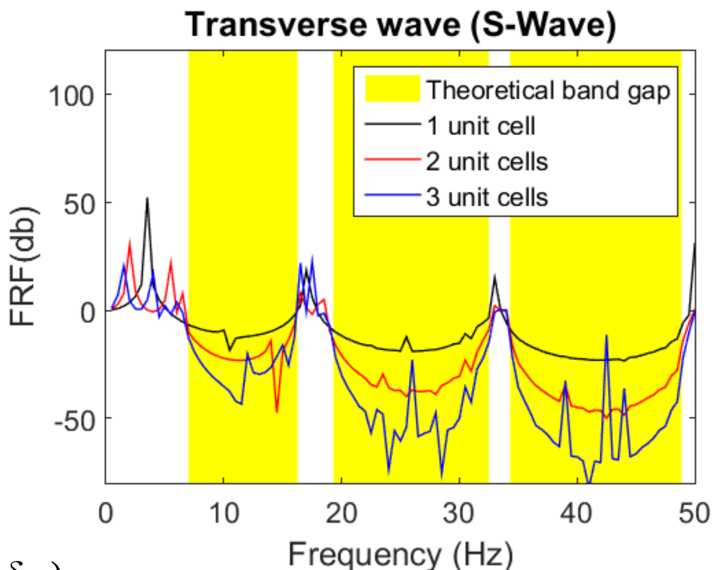


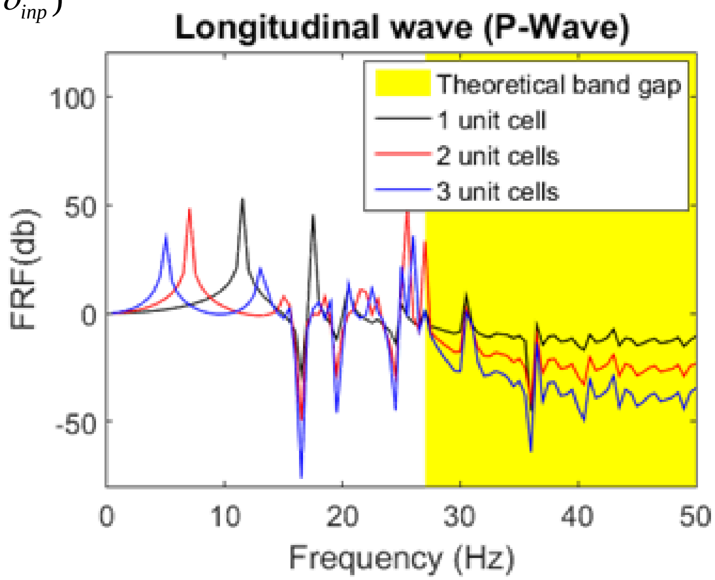
# Parametric study of 1D periodic foundations

## Effect of number of unit cells




$$FRF = 20\log(\delta_{out} / \delta_{inp})$$

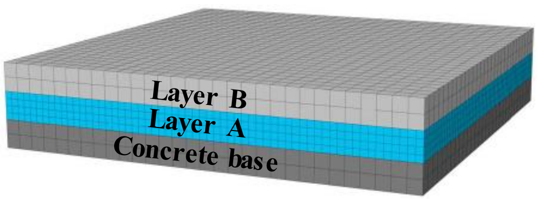


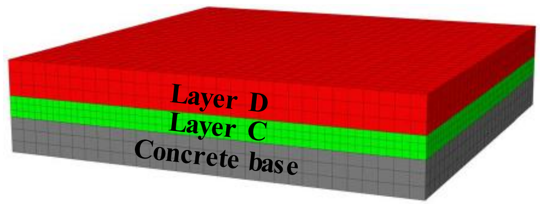
# Parametric study of 1D periodic foundations

## Effect of combined unit cells

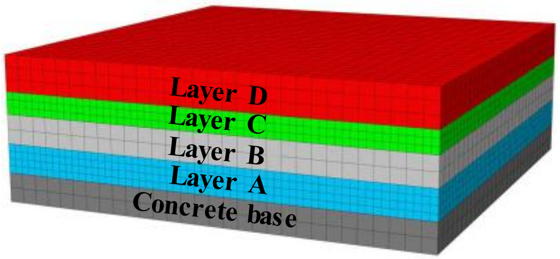
Unit cell-1

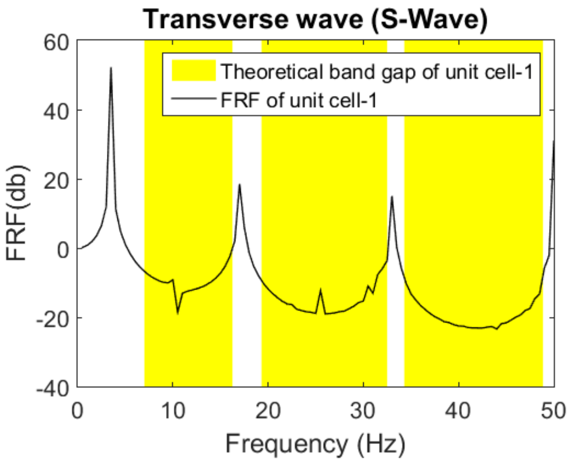


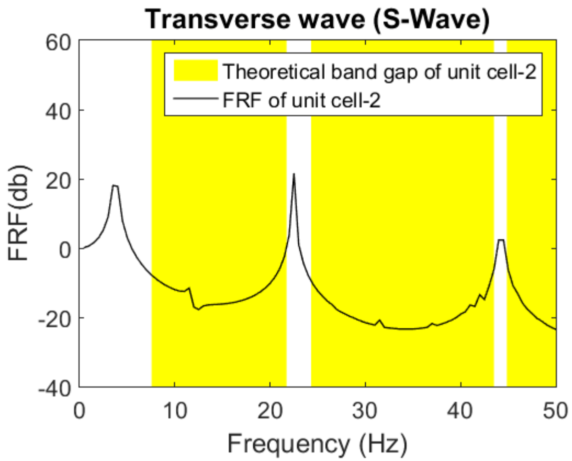

Unit cell-2

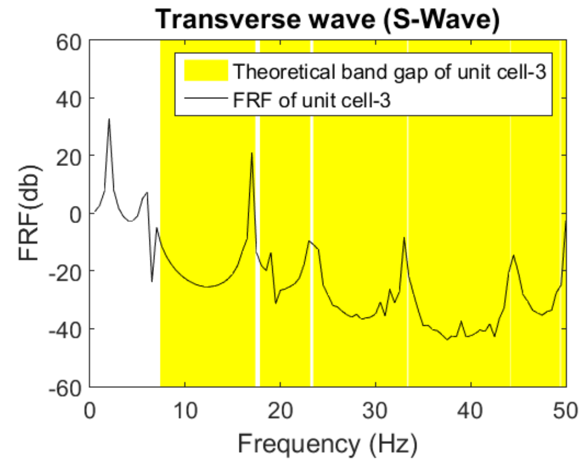



Unit cell-3



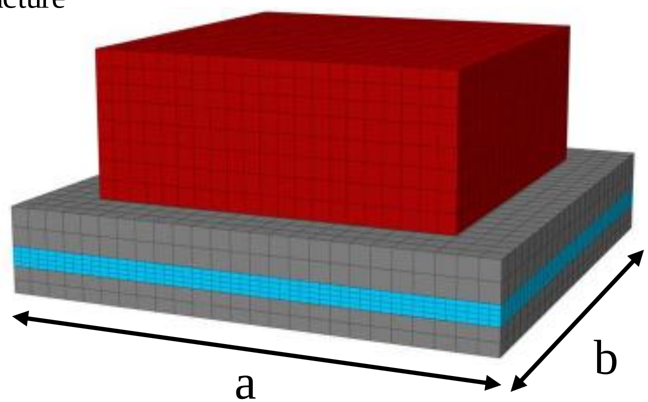



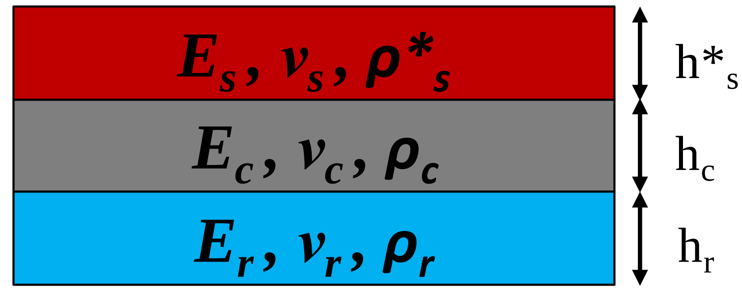
# Parametric study of 1D periodic foundations

## Effect of superstructure

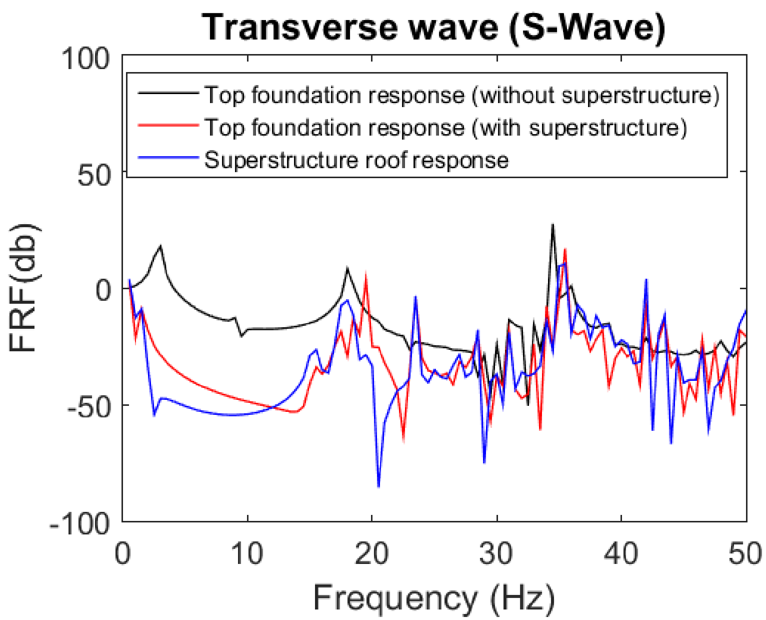
$f_{superstructure}$ =10 Hz

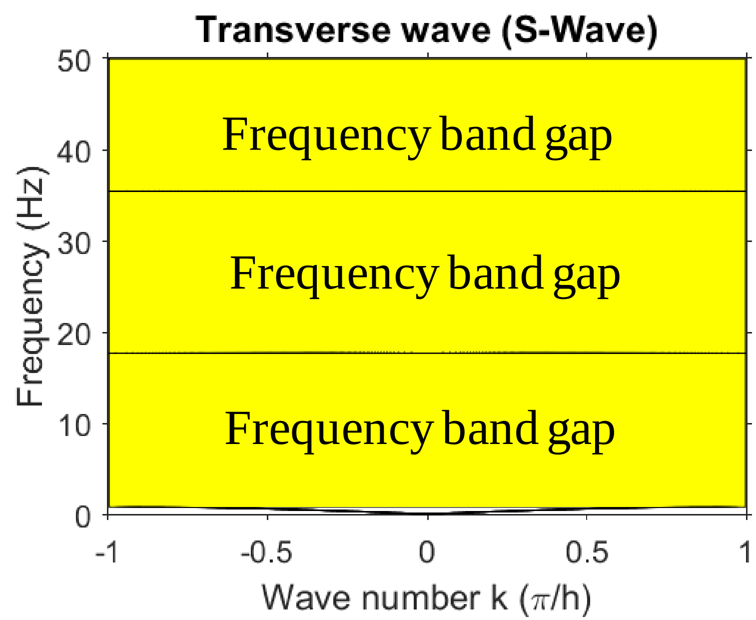



## Equivalent model




Where: $\rho^*_s = \dfrac{W_{superstructure}}{a \times b \times h^*_s}$

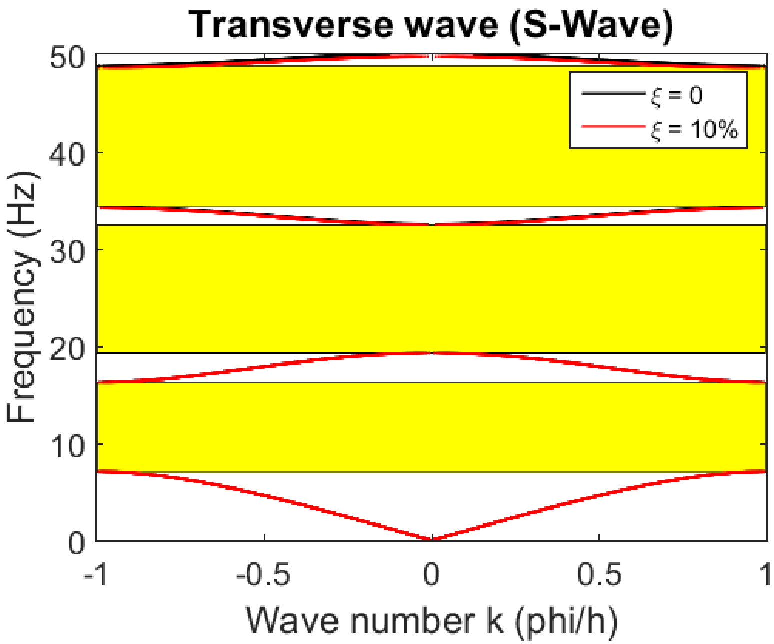


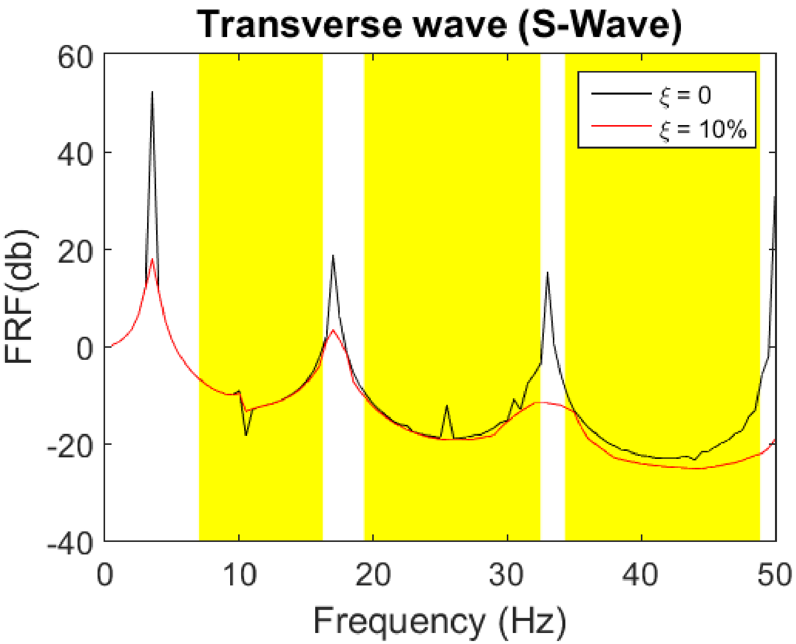



# Parametric study of 1D periodic foundations

## Effect of damping

$$\omega_d(\mathbf{K}) = \omega(\mathbf{K})\sqrt{1-\zeta(\mathbf{K})^2}$$ [1]



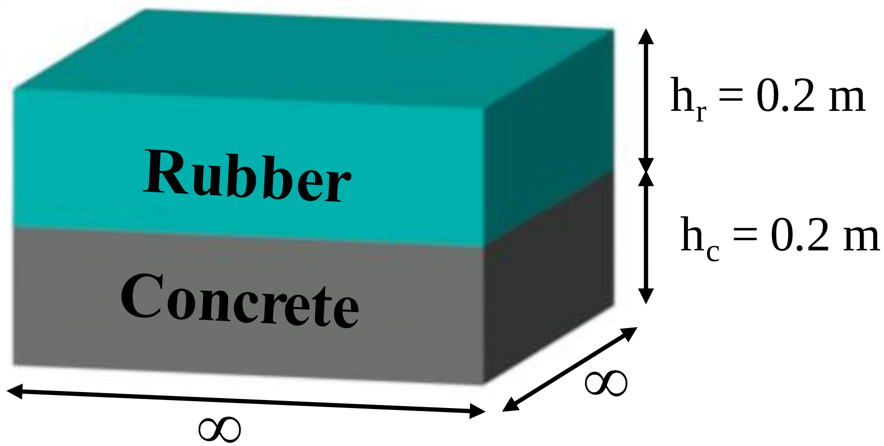


[1] Hussein, M. I. (2009). Theory of damped Bloch waves in elastic media. *Physical Review B, 80*(21), 212301.

# Design guidelines of 1D periodic foundations

**One unit cell of 1D periodic foundations**

**Fix geometric properties**



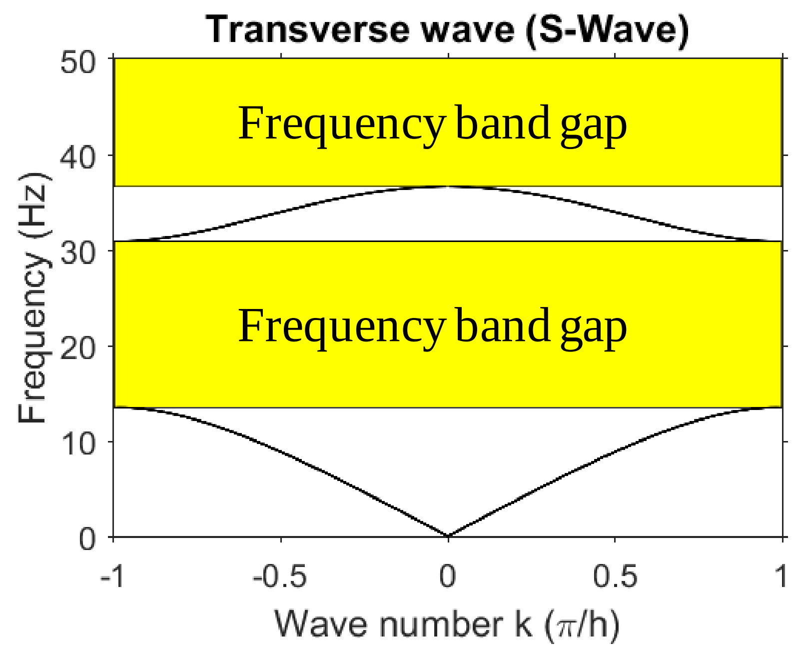

Unit cell size = 1

Rubber to concrete thickness ratio = 1

**Fix material properties**

| Material | Young's Modulus (Pa) | Density (kg/m$^3$) | Poisson's Ratio |
|---|---|---|---|
| Concrete | $3.14\times10^{10}$ | 2300 | 0.2 |
| Rubber | $5.8\times10^{5}$ | 1300 | 0.463 |

Dispersion curve for infinite number of unit cells

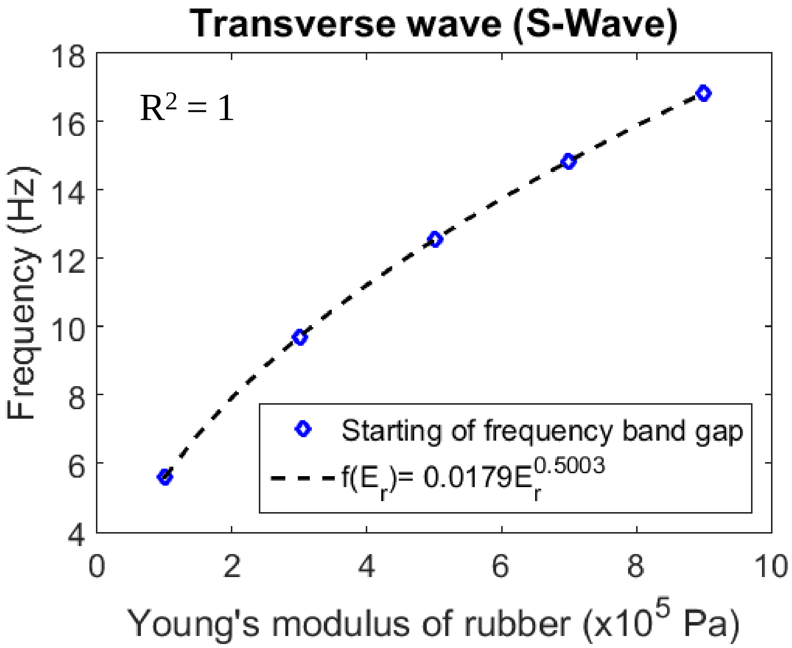


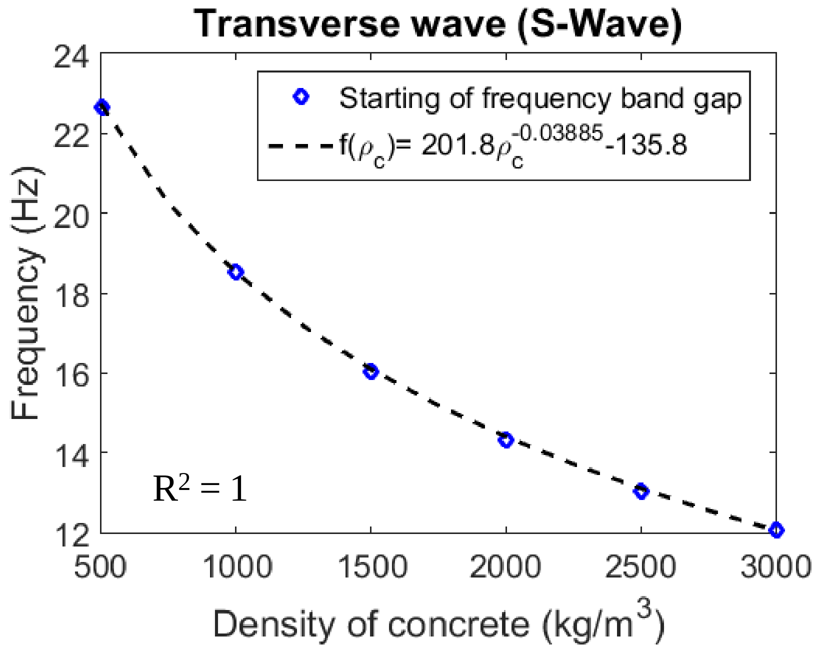

Starting of 1$^{st}$ frequency band gap = 13.51 Hz

Width of 1$^{st}$ frequency band gap = 17.36 Hz

# Design guidelines of 1D periodic foundations

Perform regression on each contributing factor




Normalized by the starting of frequency band gap from fixed property

Function of Young's modulus of rubber $F_1(\mathrm{E}_r) = \dfrac{0.01769\,\mathrm{E}_r^{\;0.5003}}{13.51} = 1.3094\times 10^{-3}\,\mathrm{E}_r^{\;0.5003}$

Function of density of concrete $F_4(\rho_c) = \dfrac{201.8\rho_c^{\;-0.03885} - 135.8}{13.51} = 14.937\rho_c^{\;-0.03885} - 10.0518$

# Design guidelines of 1D periodic foundations

**S-Wave design parameter**

| **Parameter** | **Function** |
|---|---|
| Young's modulus of rubber ($E_r$) | $F_1(E_r) = 1.3094 \times 10^{-3} E_r^{0.5003}$ |
| Density of rubber ($\rho_r$) | $F_2(\rho_r) = \left(2.814\rho_r + 1.627 \times 10^5\right) / \left(13.51\rho_r + 13.6451 \times 10^4\right)$ |
| Poisson's ratio of rubber ($\nu_r$) | $F_3(\nu_r) = -0.4139\nu_r^{0.6263} + 1.2561$ |
| Density of concrete ($\rho_c$) | $F_4(\rho_c) = 14.937\rho_c^{-0.03885} - 10.0518$ |
| Unit cell size (S) | $F_5(S) = 0.4 / S$ |
| Rubber to concrete thickness ratio (r) | $F_6(r) = 0.6403e^{-2.878r} + 0.9489e^{0.01594r}$ |

Starting of frequency band gap = $13.51 F_1(E_r) F_2(\rho_r) F_3(\nu_r) F_4(\rho_c) F_5(S) F_6(r)$

# Design guidelines of 1D periodic foundations

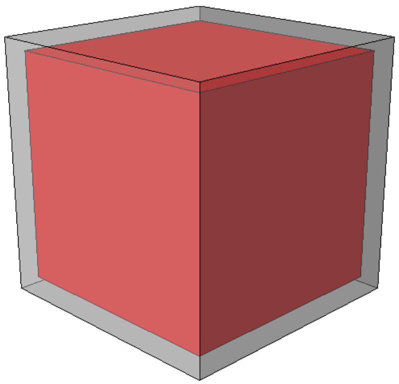
**S-Wave design parameters**

| **Parameter** | **Function** |
|---|---|
| Young's modulus of rubber ($E_r$) | $G_1(E_r) = 1.3185\times10^{-3}\,E_r^{\,0.4996}$ |
| Density of rubber ($\rho_r$) | $G_2(\rho_r) = 98.0991\rho_r^{\,-0.5964} - 0.3632$ |
| Poisson's ratio of rubber ($\nu_r$) | $G_3(\nu_r) = -0.4112\nu_r^{\,0.6325} + 1.2523$ |
| Density of concrete ($\rho_c$) | $G_4(\rho_c) = -11.6244\rho_c^{\,-0.03885} + 9.6025$ |
| Unit cell size (S) | $G_5(S) = 0.4 / S$ |
| Rubber to concrete thickness ratio (r) | $G_6(r) = r^{-0.8319}$ |

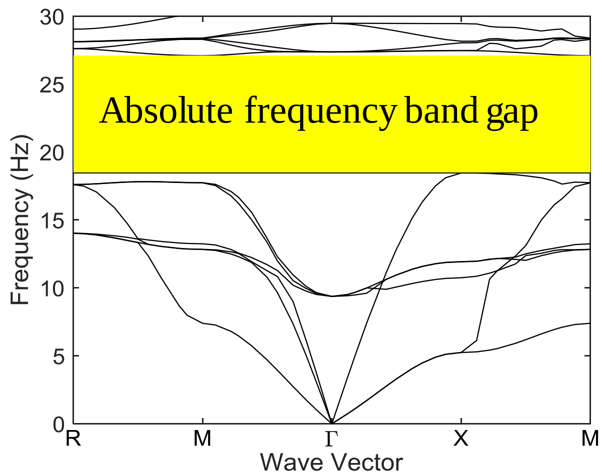
Width of frequency band gap = $17.36G_1(E_r)G_2(\rho_r)G_3(\nu_r)G_4(\rho_c)G_5(S)G_6(r)$

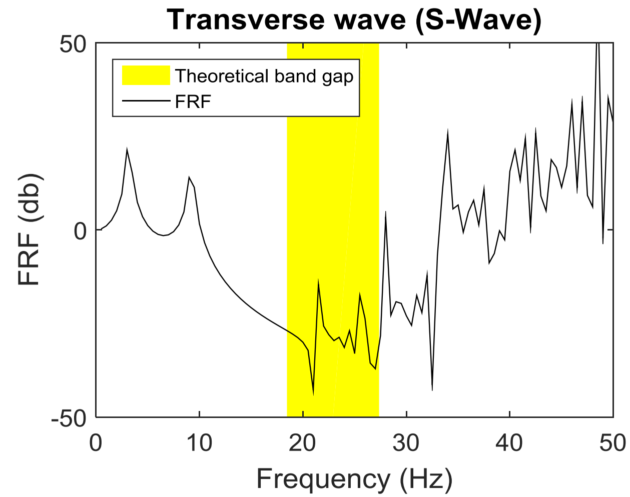
# 3D Periodic Foundations

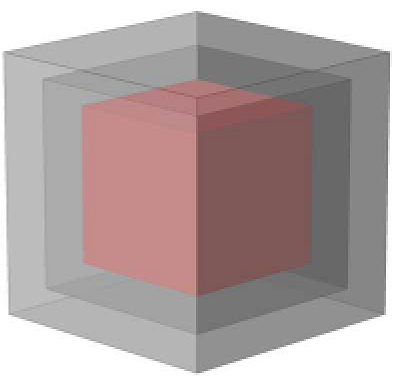
Types of unit cell in 3D periodic foundation



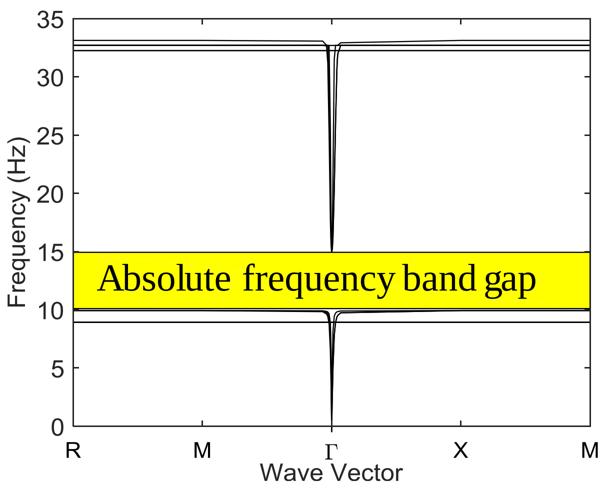
Two components

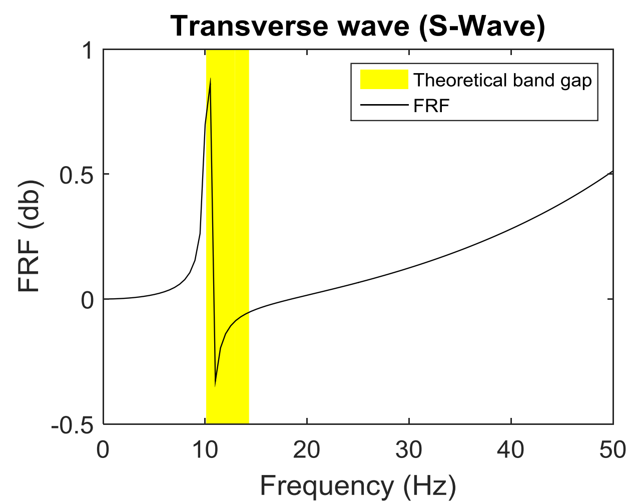


Three components



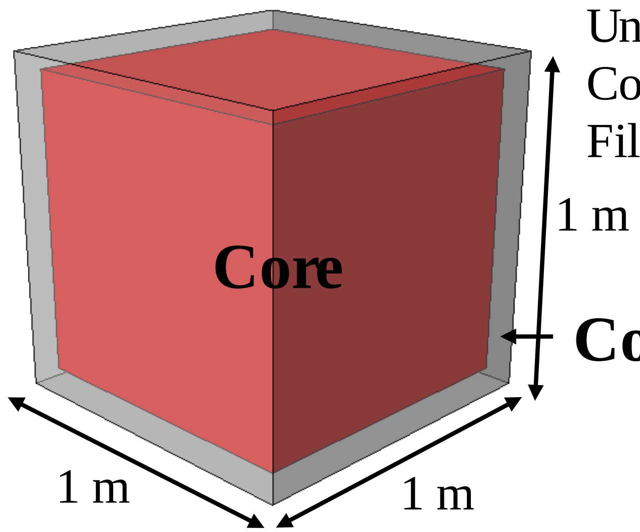



# 3D Periodic Foundations

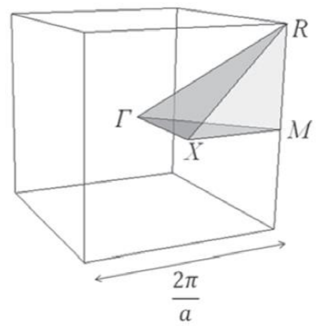
## One unit cell of two-component 3D periodic foundation

### Fix geometric properties

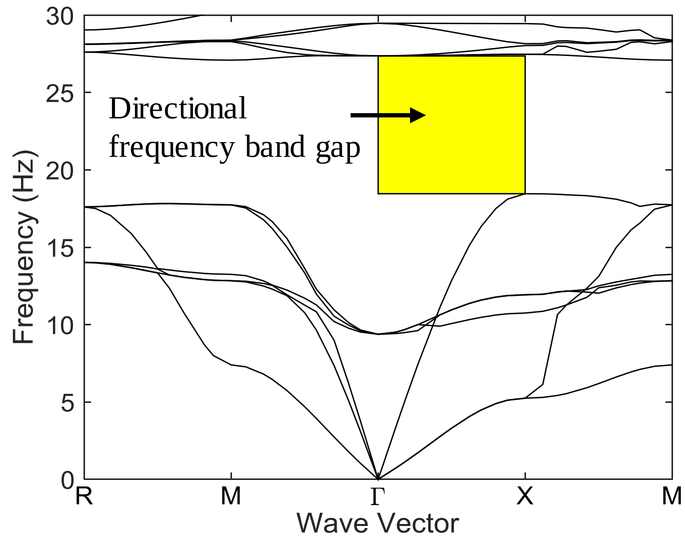
Unit cell size = 1 m
Core size = 0.9 m
Filling ratio = 0.729




### First irreducible Brillouin zone




### Dispersion curve for infinite number of unit cells

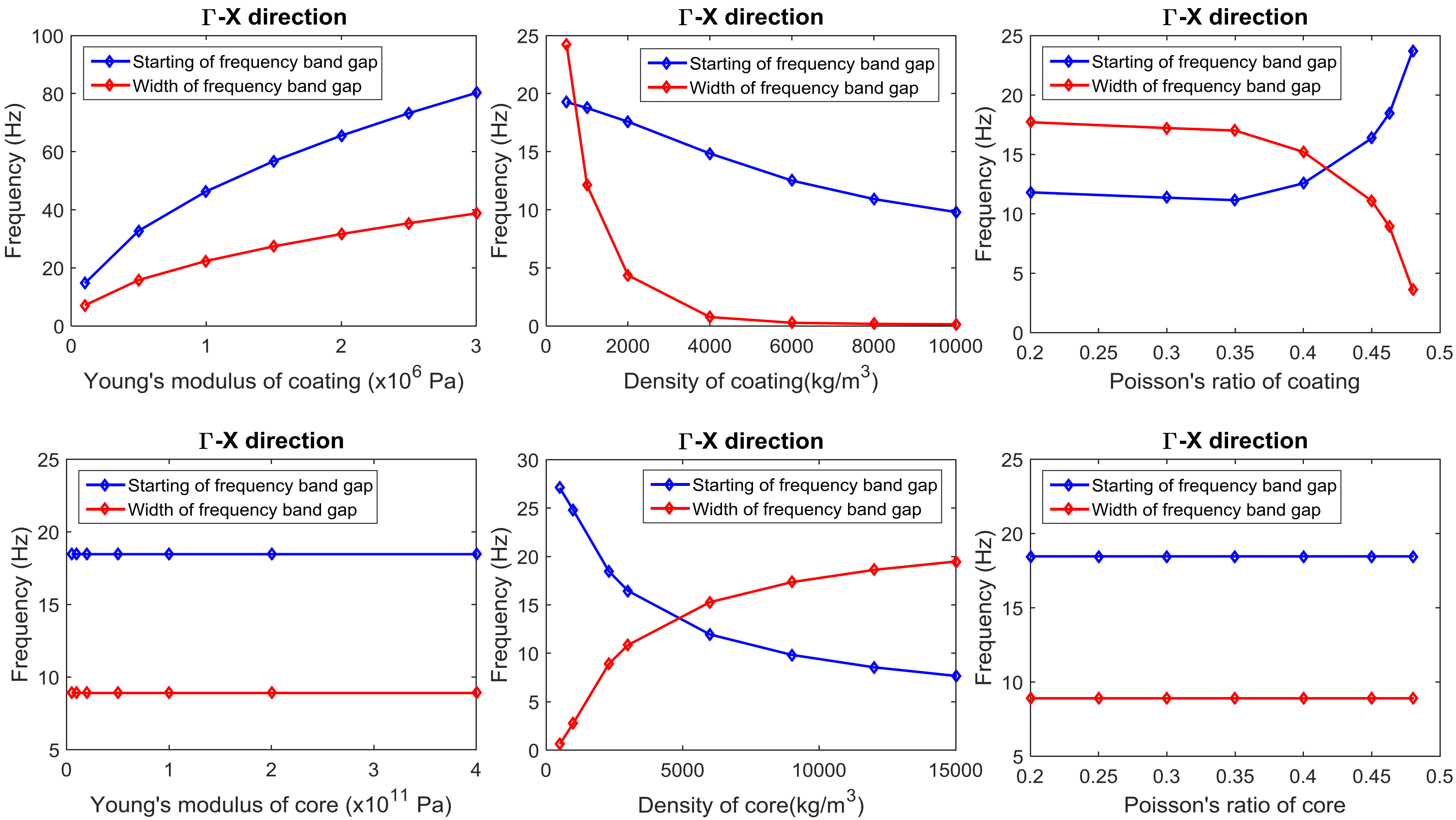



### Fix material properties

| Component | Young's Modulus (Pa) | Density (kg/m$^3$) | Poisson's Ratio |
|---|---|---|---|
| Core | $4\times10^{10}$ | 2300 | 0.2 |
| Coating | $1.586\times10^{5}$ | 1277 | 0.463 |

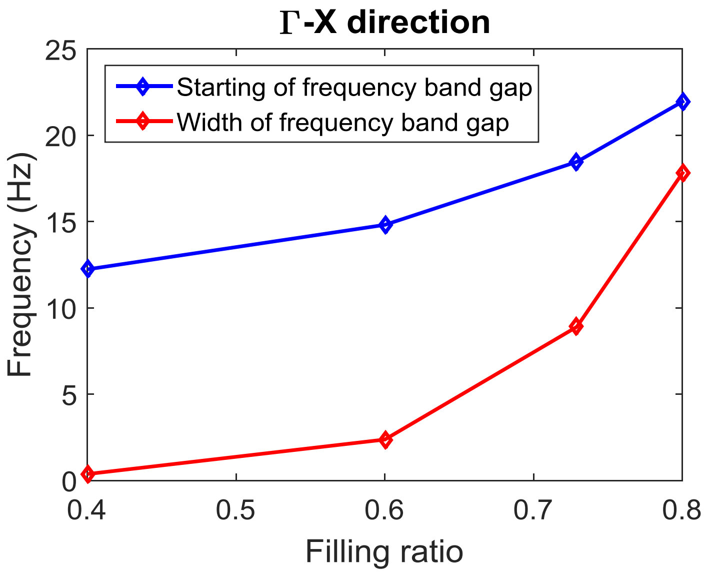
# Parametric study of 3D periodic foundations

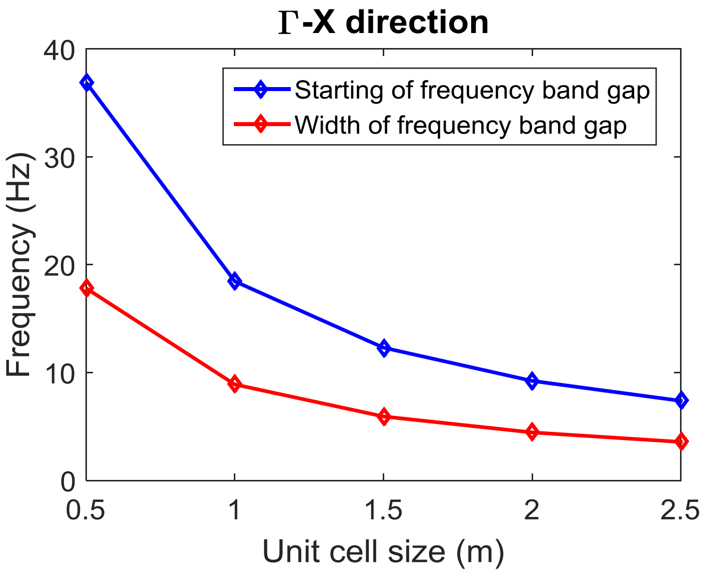
## Effect of material properties on the first directional frequency band gap



# Parametric study of 3D periodic foundations

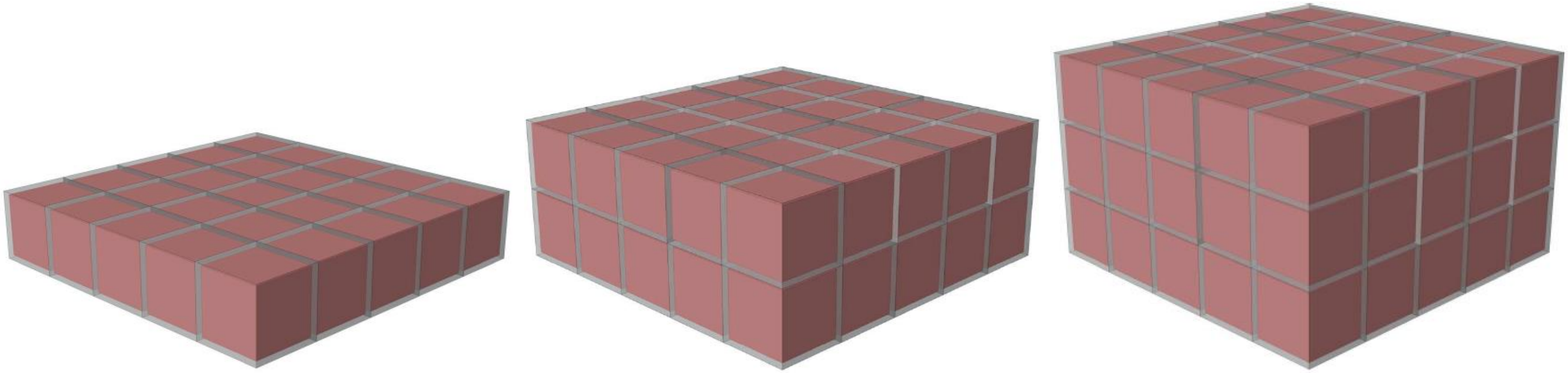
## Effect of geometric properties on the first directional frequency band gap



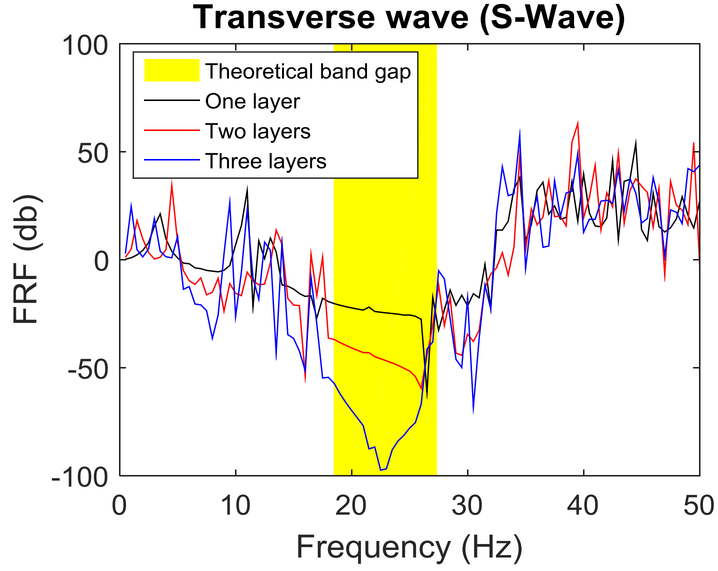



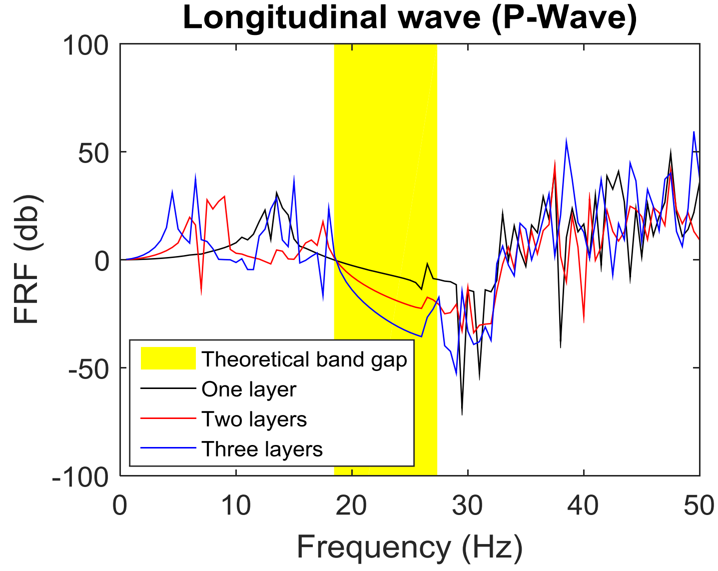
# Parametric study of 3D periodic foundation

## Effect of number of layer in vertical direction



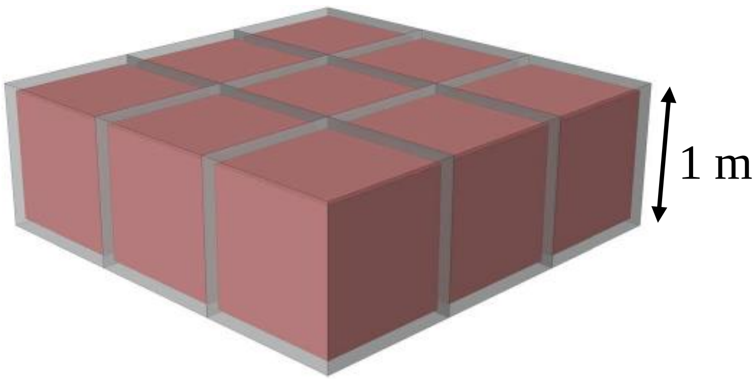
(a) One layer (b) Two layers (b) Three layers



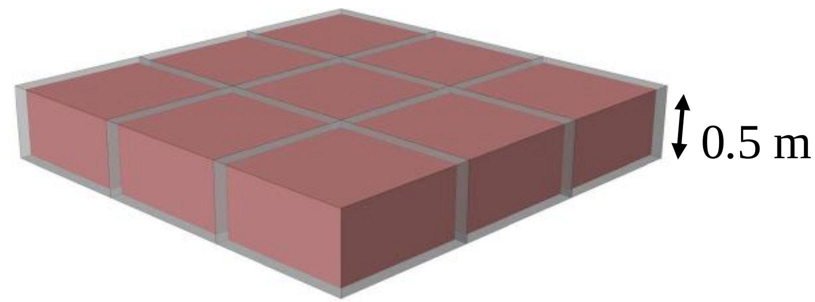



# Parametric study of 3D periodic foundations

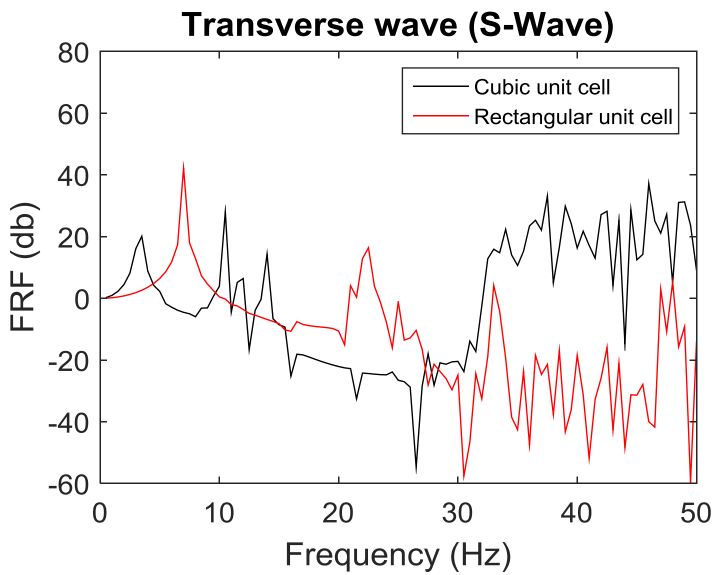
## Effect of suppressed unit cell

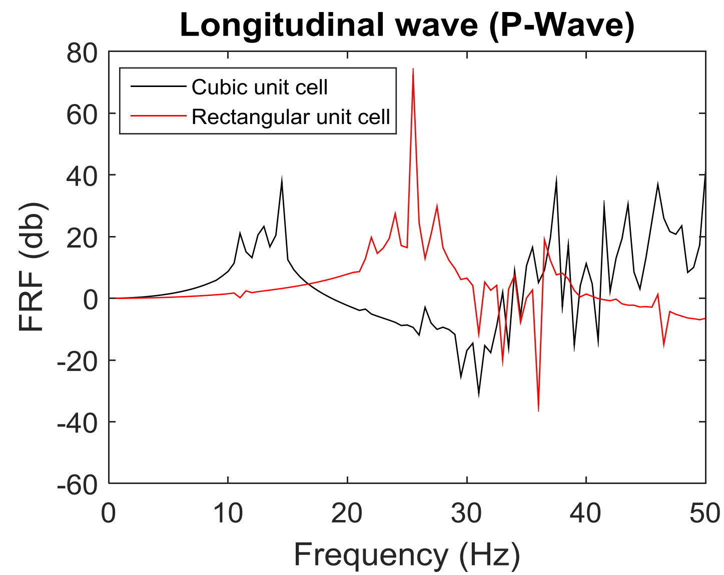



(a) Cubic unit cell




(b) Rectangular unit cell

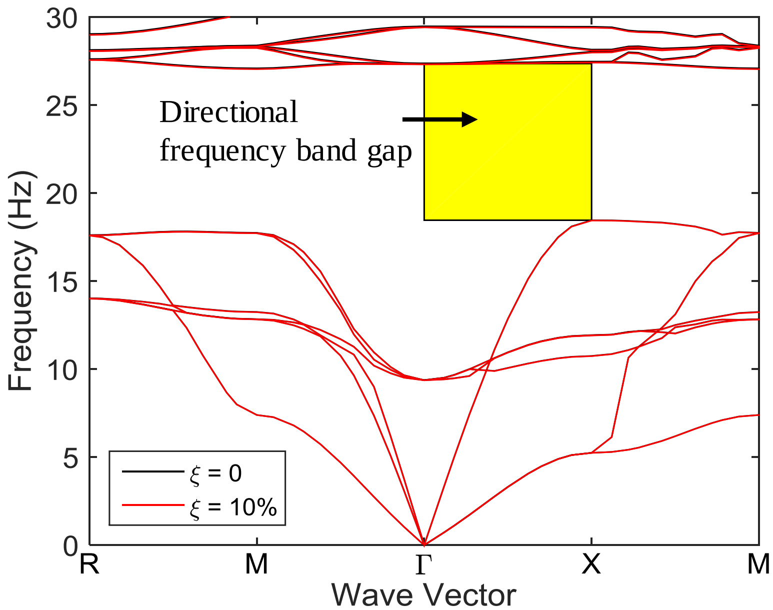


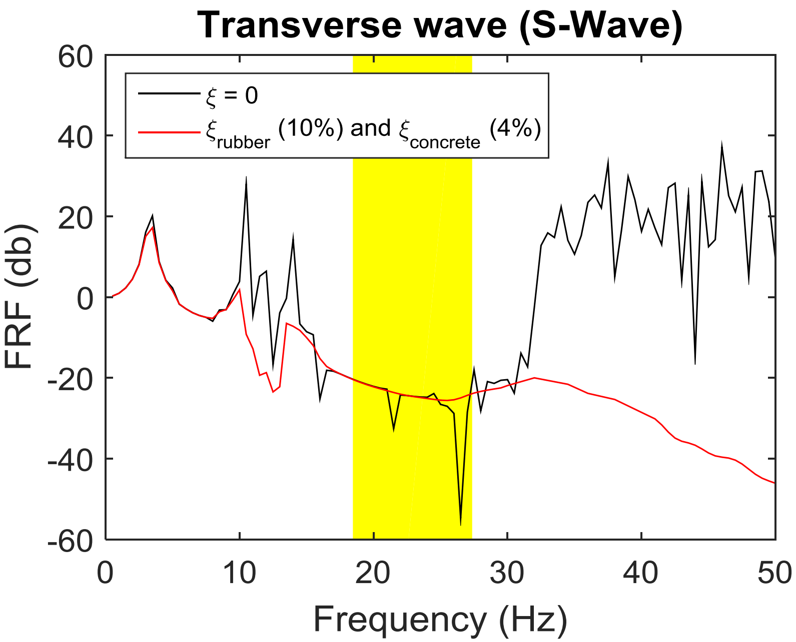



# Parametric study of 3D periodic foundations

## Effect of damping

$$\omega_d(\mathbf{K}) = \omega(\mathbf{K})\sqrt{1-\zeta(\mathbf{K})^2}$$ [1]



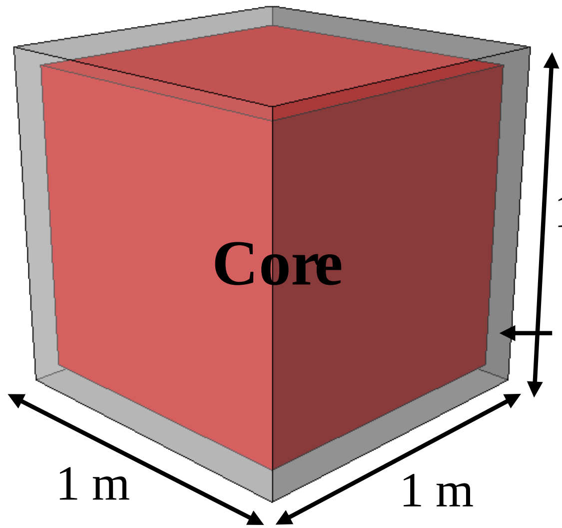




[1] Hussein, M. I. (2009). Theory of damped Bloch waves in elastic media. *Physical Review B, 80*(21), 212301.

# Design guidelines of 3D periodic foundations

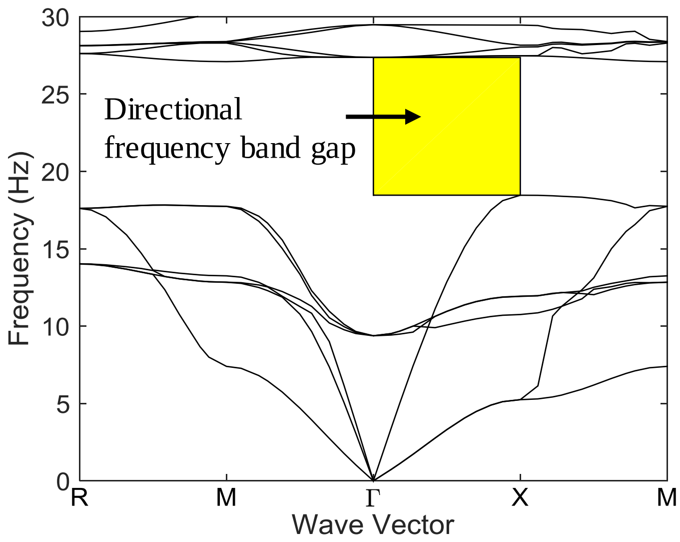
**One unit cell of two components 3D periodic foundation**

**Fix geometric properties**




Unit cell size = 1 m
Core size = 0.9 m
Filling ratio = 0.729

**Dispersion curve for infinite number of unit cells**



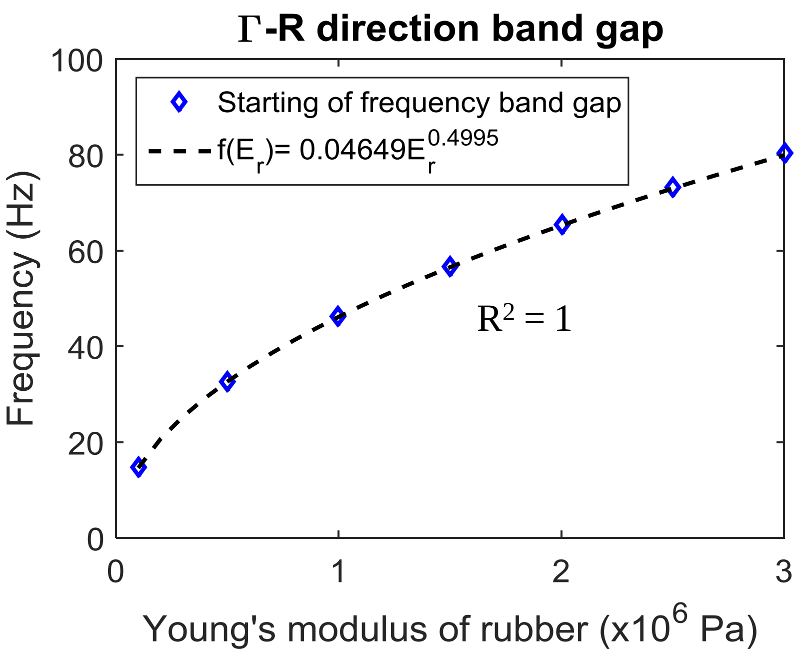

**Fix material properties**

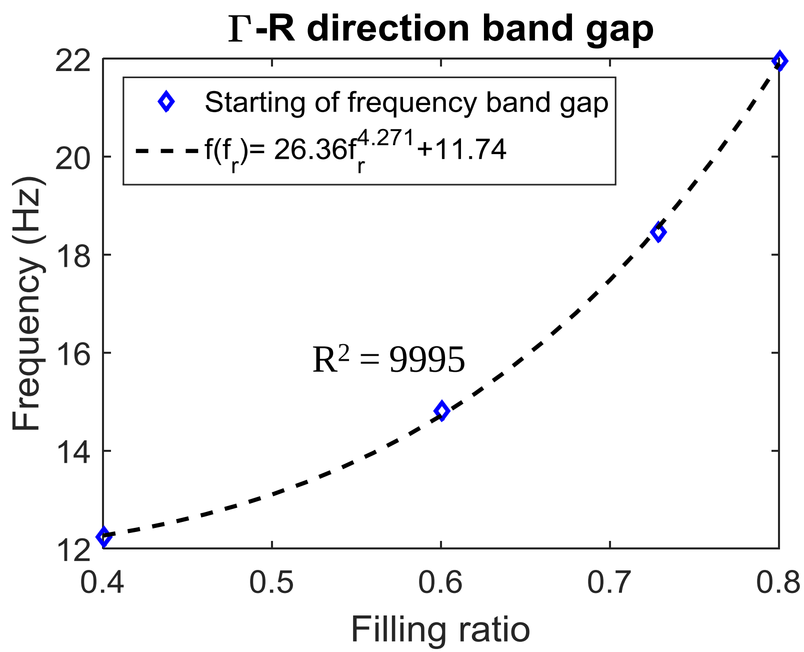
| Component | Young's Modulus (Pa) | Density (kg/m$^3$) | Poisson's Ratio |
|---|---|---|---|
| Core | $4\times10^{10}$ | 2300 | 0.2 |
| Coating | $1.586\times10^{5}$ | 1277 | 0.463 |

Starting of 1st frequency band gap = 18.46 Hz
Width of 1st frequency band gap = 8.9 Hz

# Design guidelines of 3D periodic foundations

Perform regression on each contributing factor




Normalized by the starting of frequency band gap from fixed property

Function of Young's modulus of rubber $J_1(\mathrm{E}_r) = \dfrac{0.04649\,\mathrm{E}_r^{\;0.4995}}{18.46} = 2.5184\times10^{-3}\,\mathrm{E}_r^{\;0.4995}$

Function of filling ratio $J_6(\mathrm{f_r}) = \dfrac{26.36\mathrm{f_r}^{\;4.271} + 11.74}{18.46} = 1.428\mathrm{f_r}^{\;4.271} + 0.636$

# Design guidelines of 3D periodic foundations

**Design parameter**

| **Parameter** | **Function** |
|---|---|
| Young's modulus of rubber ($E_r$) | $J_1(E_r) = 2.5184\times10^{-3}\, E_r^{0.4995}$ |
| Density of rubber ($\rho_r$) | $J_2(\rho_r) = 0.9 + 0.1793\cos(0.000187\rho_r) - 0.3282\sin(0.000187\rho_r)$ |
| Poisson's ratio of rubber ($\nu_r$) | $J_3(\nu_r) = 0.688e^{-0.3911\nu_r} + 9.6479\times10^{-7}e^{28.14\nu_r}$ |
| Density of concrete ($\rho_c$) | $J_4(\rho_c) = 0.9832e^{-0.0004377\rho_c} + 0.7053e^{-3.557\times10^{-5}\rho_c}$ |
| Unit cell size (S) | $J_5(S) = 1/S$ |
| Filling ratio ($f_r$) | $J_6(f_r) = 1.428 f_r^{4.271} + 0.636$ |

Starting of directional frequency band gap = $18.46J_1(E_r)J_2(\rho_r)J_3(\nu_r)J_4(\rho_c)J_5(S)J_6(f_r)$

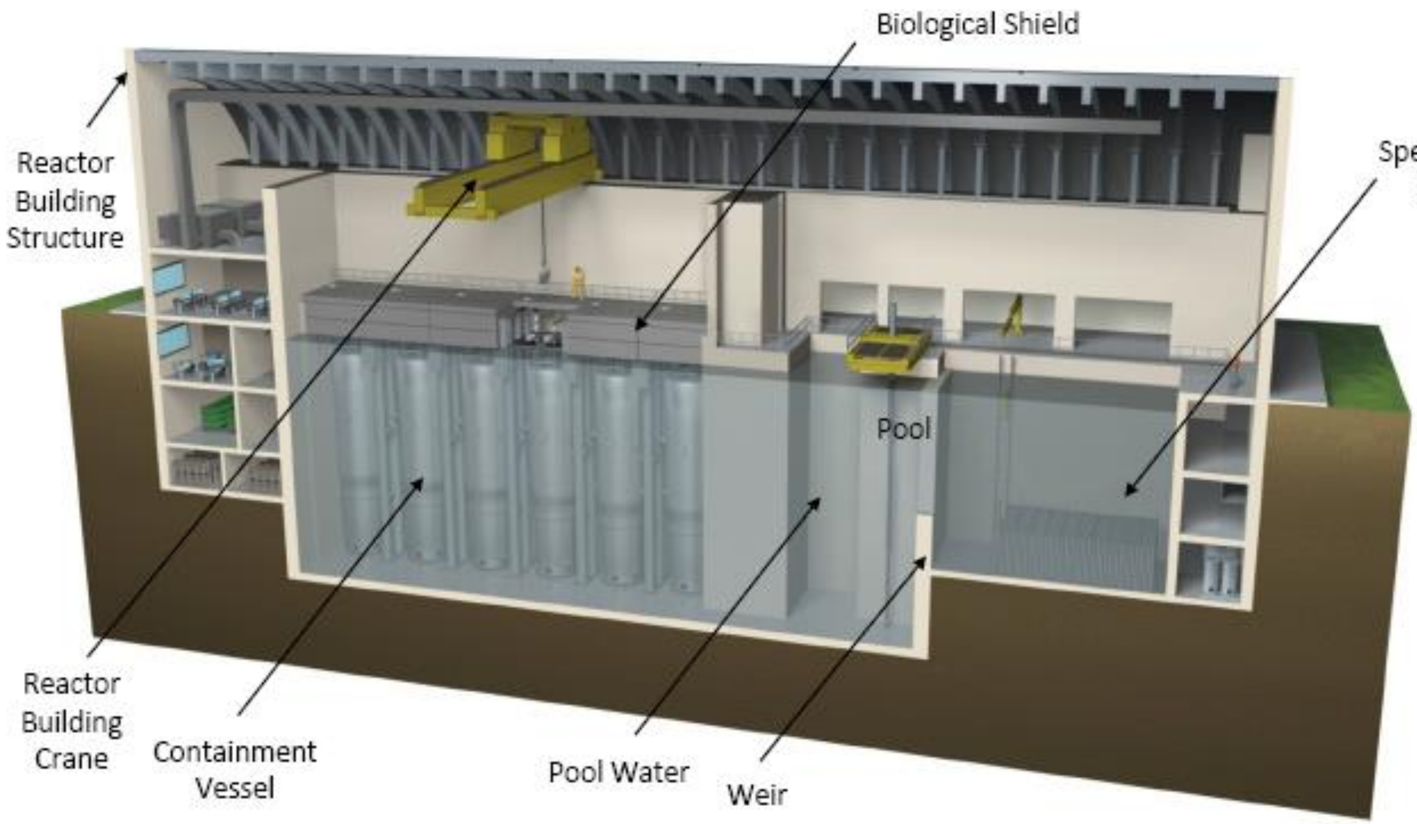
# Design guidelines of 3D periodic foundations

**Design parameter**

| **Parameter** | **Function** |
|---|---|
| Young's modulus of rubber ($E_r$) | $K_1(E_r) = 2.51\times10^{-3} E_r^{\,0.5001}$ |
| Density of rubber ($\rho_r$) | $K_2(\rho_r) = \left(0.0003842\rho_r^{\,2} - 5.454\rho_r + 19290\right) / \left(8.9\rho_r + 1661.63\right)$ |
| Poisson's ratio of rubber ($\nu_r$) | $K_3(\nu_r) = -8488.764\nu_r^{\,11.76} + 1.9472$ |
| Density of concrete ($\rho_c$) | $K_4(\rho_c) = 1.7506e^{1.512\times10^{-5}\rho_c} - 2.1157e^{-0.0004053\rho_c}$ |
| Unit cell size (S) | $K_5(S) = 1/S$ |
| Filling ratio ($f_r$) | $K_6(f_r) = 10.064 f_r^{\,7.252}$ |

Width of directional frequency band gap = $8.9K_1(E_r)K_2(\rho_r)K_3(\nu_r)K_4(\rho_c)K_5(S)K_6(f_r)$

# NuScale Reactor Building



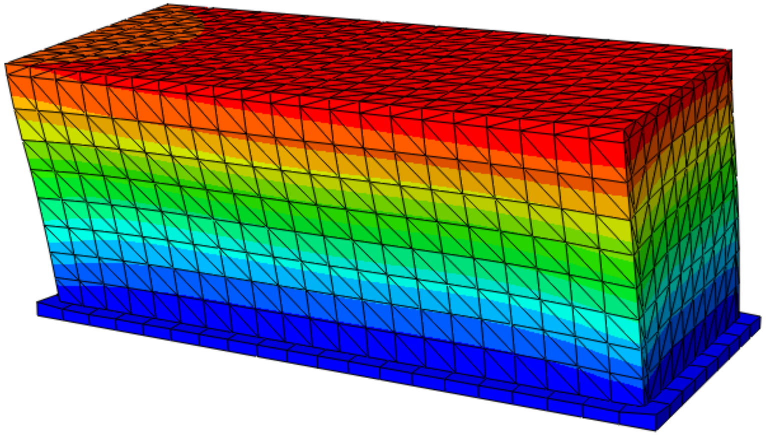
# Finite element model of reactor building

Nuclear reactor building is made of reinforced concrete.

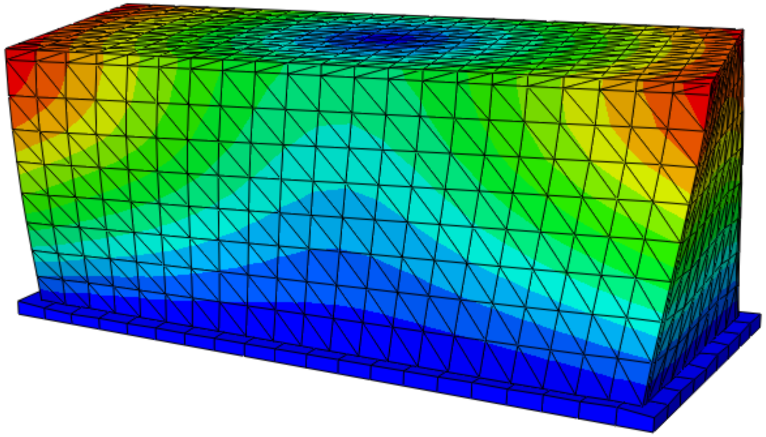
Superimposed dead load:

- Water in the reactor pool = 7 million gallon
- Crane + utilities = 800 ton
- Small modular reactors = 12@800 ton

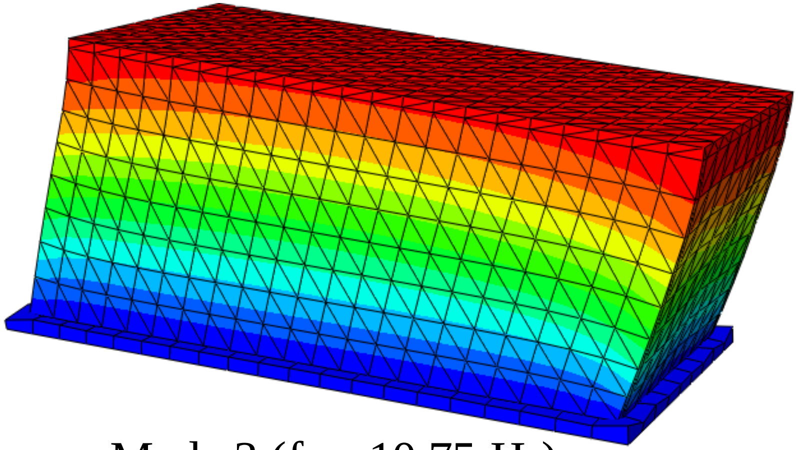
# Finite element model of reactor building



Mode 1 ($f_n$ = 6.13 Hz)



Mode 2 ($f_n$ = 10 Hz)



Mode 3 ($f_n$ = 10.75 Hz)

# Conclusions

- Basic theory of periodic foundations have been understood.
- Behavior of 1D and 3D periodic foundations have been critically examined.
- Simplified design guidelines for 1D and 3D periodic foundations have been proposed.
- Simplified drawing of reactor building has been obtained from NuScale Power.
- Project will proceed on schedule.

# Thank you.