Attribute-Preserving Optimal Network Reductions

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Resources for the Future

Context

- Objective: Develop reduced network equivalencing procedures that preserve certain attributes.
- Reduced network equivalents have been used:
  - Speed execution of problems
  - Size problems to available computation resources.
    - E4ST Application
    - Dynamic simulations, etc.
- Traditional network reductions only preserve certain structures
  - Ward reduction
    - Preserves nodal voltages, and branch flows for base case only under linearity assumption.
  - The improved Ward (e.g. PV-Ward or extended Ward)
    - Gives better performance on matching reactive support.
  - REI
    - Reactive support better modeled.
    - Hot start method which can preserve base case power flow solutions (bus voltage, branch flow, etc.).
    - Inaccurate when operating condition changes.
- Objective: Targeted network reductions.
- Benefits: Allow more accurate simulations of electric power networks.

Scope

- Developing attribute-preserving network equivalents.
  - Topology
  - Branch values
  - Generator placement
  - Load models
- Reduced dc equivalents that preserve branch flow values.
  - Finding optimal branch reactances for ac-to-dc model conversion
  - Bus aggregation
  - Ward-type reduction
  - Generalized optimization formulation for dc equivalents
- This past cycle looked at:
  - Generalized optimization-based Ward-type reduction formulation applied large dc systems.
  - Applied optimal generator placement in reductions of large dc systems.
  - Reductions which preserve bus voltage values through VC in ac systems.
  - Network reduction toolbox upgrade.
  - Transmission expansion corridors.

Outline

- Optimization based Ward reduction (OP-Ward) dc systems (Yujia Zhu)
- Optimal generator placement on ERCOT, WECC and EI (Yujia)
- Network reduction toolbox upgrade (Yujia)
- Transmission expansion corridors (Team)
- Inverse function equivalents—central idea—linear case (Shruti Rao)
- Application of inverse function equivalents to (nonlinear) ac systems for bus voltage preservation (Shruti)

OP-Ward reduction

- Last year:
- We showed that the Ward and OP-Ward gave identical results for 6-bus system.
- Tested the method on a 9-bus and IEEE 118-bus systems with mixed results.
- Identified a fundamental issue causing a rank deficiency problem in some cases.

OP-Ward reduction

- Idea: Minimize the branch flow errors in the retained model portion.
- Formulate the problem as an unconstrained optimization problem:
- Objective: $$\min_{y_i} \left\| \Lambda_1 \boldsymbol{y} - \boldsymbol{b} \right\|_2 \qquad (1)$$
- where:

$$\Lambda_1 = \begin{bmatrix} PTDF_{full}^{r} \cdot C^T diag(c_1) \\ PTDF_{full}^{r} \cdot C^T diag(c_2) \\ \vdots \\ PTDF_{full}^{r} \cdot C^T diag(c_{N-1}) \end{bmatrix} \qquad \boldsymbol{b} = \begin{bmatrix} b_{f1} \\ b_{f2} \\ \vdots \\ b_{f,N-1} \end{bmatrix}$$

- C is the branch-bus incidence matrix and c_i is the ith column in C.
- b_{fi} is the ith column in the full model branch susceptance matrix.
  - N-1 is number of retained buses.

OP-Ward reduction

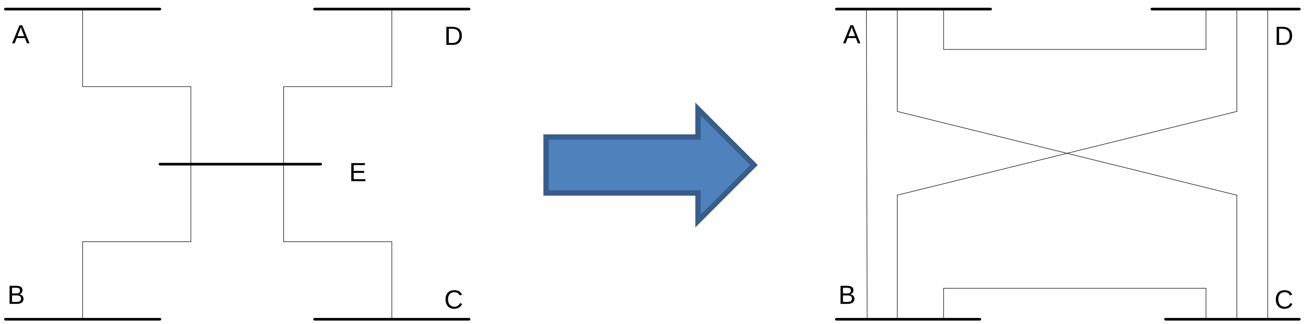
- Test cases:

Case #	Test system	# of retained buses	# of external buses
1	9-bus	7	2
2	IEEE 118-bus	88	30
3	IEEE 118-bus	68	50
4	IEEE 118-bus	35	83

- Error metric: Max branch reactance error %.
- Large errors (>50%) occurred.
- The Λ_1 matrix is rank deficient.

OP-Ward reduction

- Star-mesh conversion.



$$\Lambda_1 = \begin{bmatrix} PTDF_{full}^{r} \cdot C^T diag(c_1) \\ PTDF_{full}^{r} \cdot C^T diag(c_2) \\ \vdots \\ PTDF_{full}^{r} \cdot C^T diag(c_{N-1}) \end{bmatrix}$$

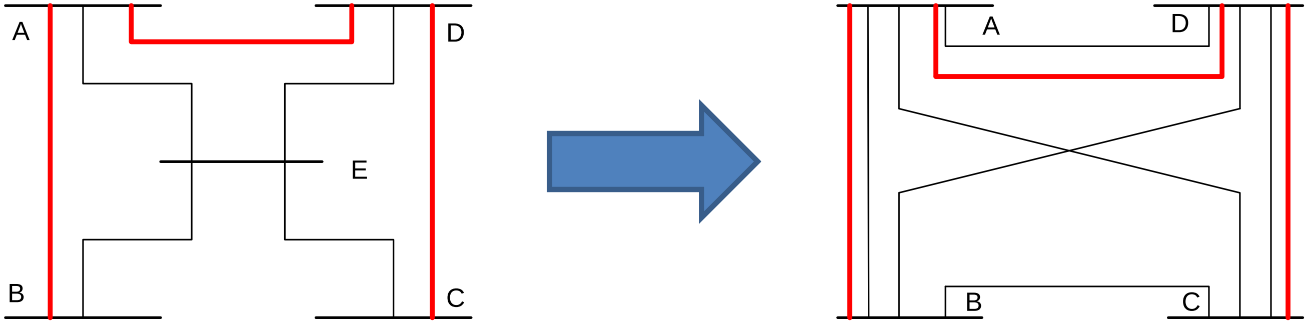
- $PTDF_{full}^{r}$ is the portion of the PTDF matrix of the full model corresponding to retained branches in the reduced model.
- In the star-mesh conversion, no branch is preserved thus the Λ_1 matrix in (1) can not be created.

OP-Ward reduction

- Curing the rank deficiency problem.
- Theory:
  - Add enough pseudo branches to full network to make the Λ_1 matrix of full rank.
  - Remove pseudo branches from the reduced model.

OP-Ward reduction

- Prior to the reduction process add three pseudo branches (red lines in the figures below) parallel to the three equivalent branches.



- The Λ_1 matrix based on the three pseudo branches is of full rank.

OP-Ward reduction

- Test results

Case #	# of rank increase	Error (%)	Problem solved?
1	1	4.21E-13	Y
2	1	2.14E-14	Y
3	5	9.26E-13	Y
4	7	3.11E-13	Y

- All cases yielded negligible errors.

OP-Ward reduction

- Heuristic rules for minimizing number of pseudo branches as follows*.
  - 1. Every bus must have either a pseudo or retained branch incident on it.
  - 2. The number of pseudo branches added in a network must be no less than the maximum number of equivalent lines incident on any bus.
- Reduced the number of pseudo branches from 338 to 21 in Test Case #4 while retaining a small maximum error (6.3E-11% v. 3.1E-13%).

*Assuming radial buses and loops were properly handled.

Generator Placement

- Last year:
- Tested three generator placement methods on small systems:
- Shortest Electrical Distance (SED) based method: place the external generator at a retained generator bus which is closest to its original location in terms of electrical distance.
- Optimization based Generator Placement (OGP) method: place the external generators by solving an mixed integer linear programming problem whose objective is minimizing generation cost while retaining congestion status within the system.

Generator Placement

- Minimum Shift Factor Change (Min-SF) based method: place the external generator at the retained generator bus which has the most similar shift factor to the original external generator bus.
- In the test results we showed last year on small systems, we found that the Min-SF method is the most robust and more accurate than the OGP method.
- We tested the Min-SF and the SED methods on ERCOT, WECC and EI*.

* Tests on EI system in progress.

Generator Placement

- Two metrics were used
  - Average LMP error
  - Error in Average Energy Cost (AEC=Total $/MWh)
- Error Calculation
  - Average LMP error ($/MWh)

$$Err_{LMP} = \frac{1}{N_i}\sum_i \left(\left| LMP^i_{full} - LMP^i_{reduced} \right| \right)$$

  - Average energy cost (AEC) error ($/MWh)

$$Err_{AEC} = \left| AEC_{full} - AEC_{reduced} \right|$$

Where:

i is the index of retained buses

N_i is the number of retained buses

Generator Placement

- Baseline LMP and AEC values taken as the dc OPF results for the unreduced model.
- Compared with dc OPF results for the reduced model with generators place by:
  - SED method
  - Min-SF method

Generator Placement

- Loading scenarios generated for large systems by uniformly scaling the loads across the system.
- Only the scenarios in which the unreduced model yielded feasible dc OPF results were considered.

Generator Placement

- Statistics of the three interconnections

Full model statistics

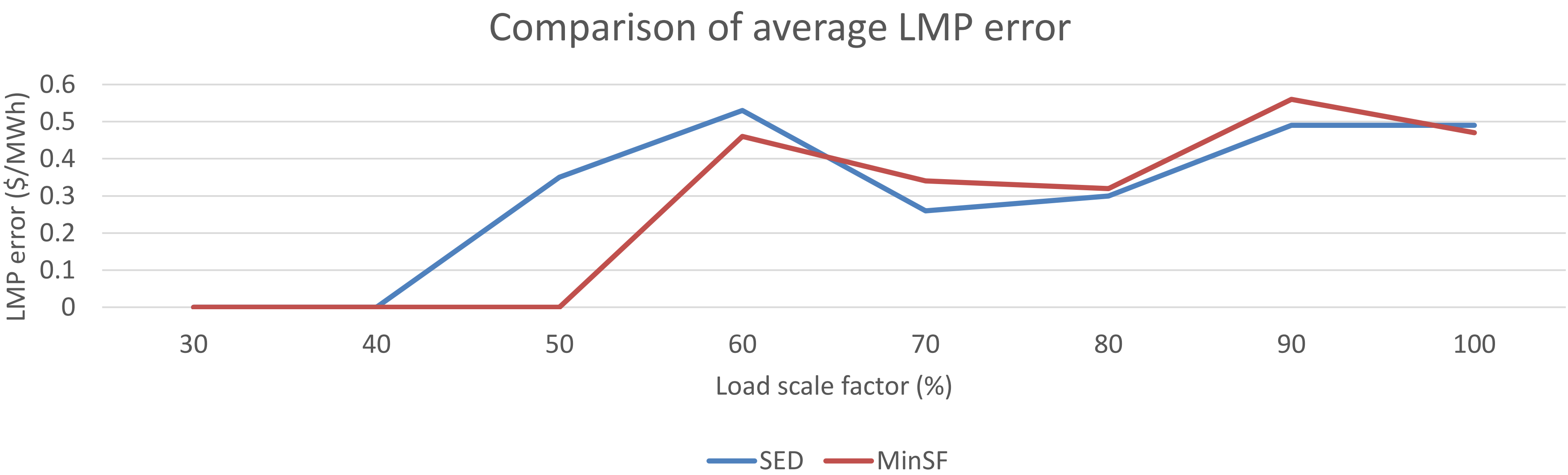
	ERCOT	WECC	EI
# of bus in full model	5633	16994	59740
# of branches in full model	7053	21539	76877
# of generators	687	3346	8190

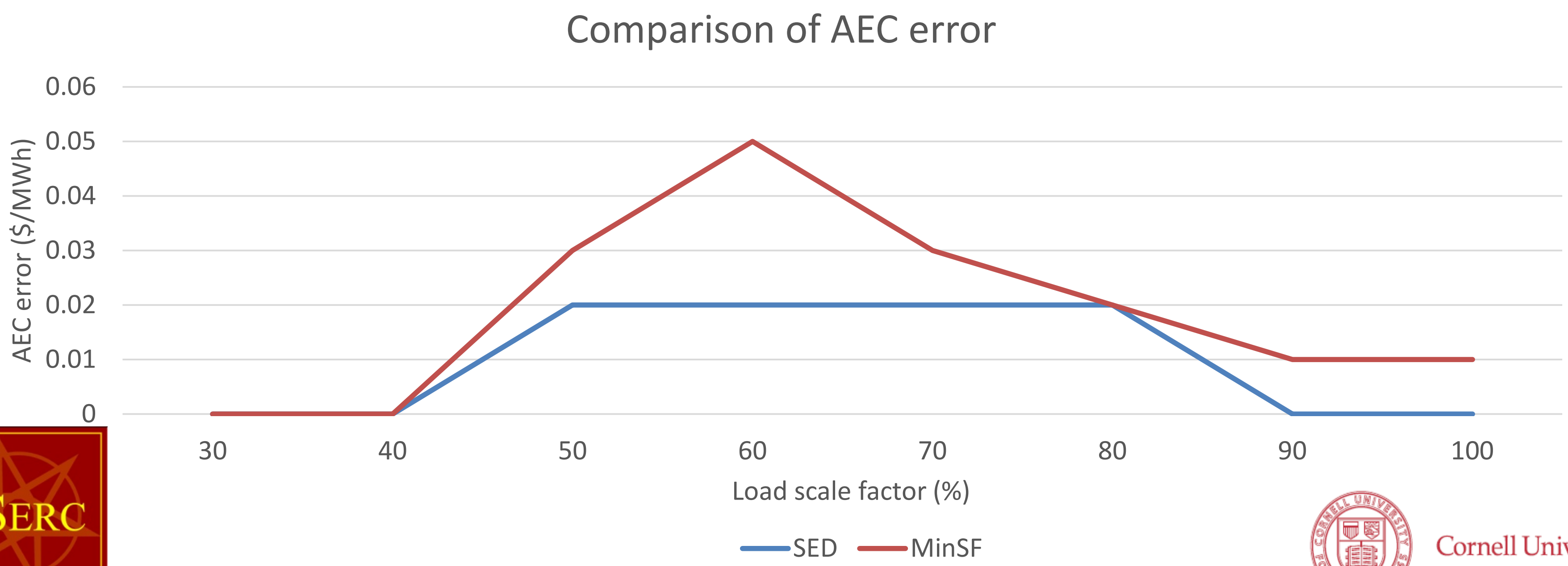
Reduced models statistics

	# of buses in less aggressive reduced model	Reduction percentage (%)	# of buses in more aggressive reduced model	Reduction percentage (%)	# of branches in non-aggressive reduced model	Reduction percentage (%)	# of branches in aggressive reduced model	Reduction percentage (%)
ERCOT	3025	53.7	389	6.91	6385	90.5	1658	23.5
WECC	6851	40.3	2305	13.6	14162	57.7	4557	21.2

Generator Placement

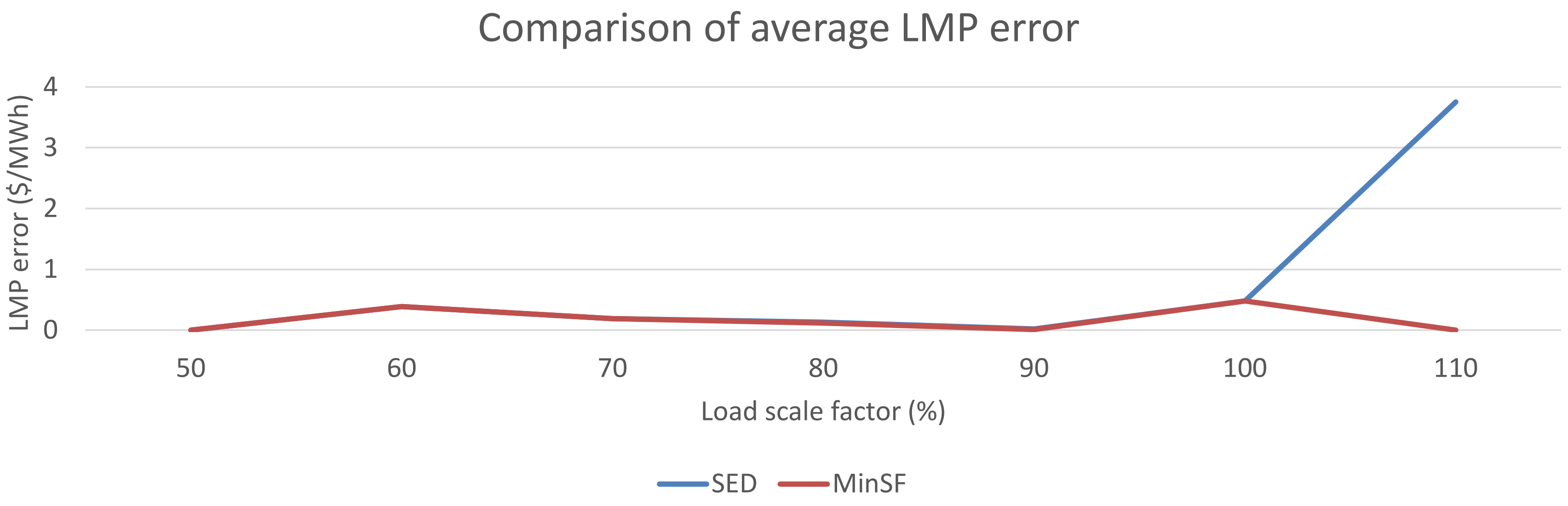
Results of WECC (6851 bus system—less aggressive≈50%)

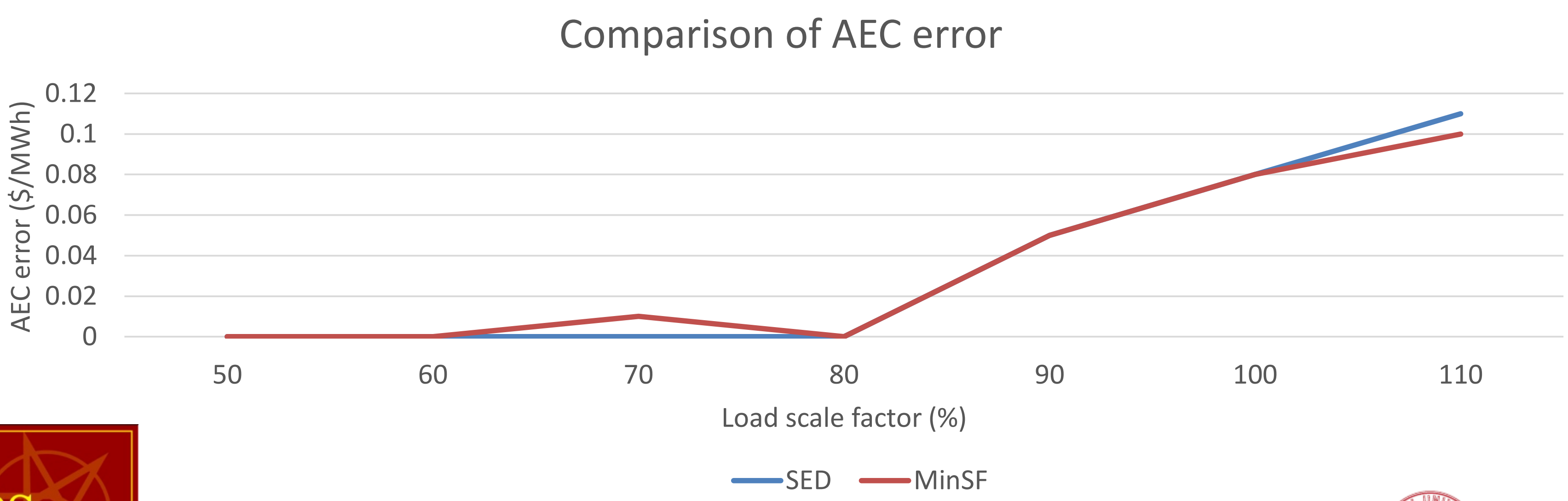




Generator Placement

Results of ERCOT (3025 bus system—less aggressive ≈50%)



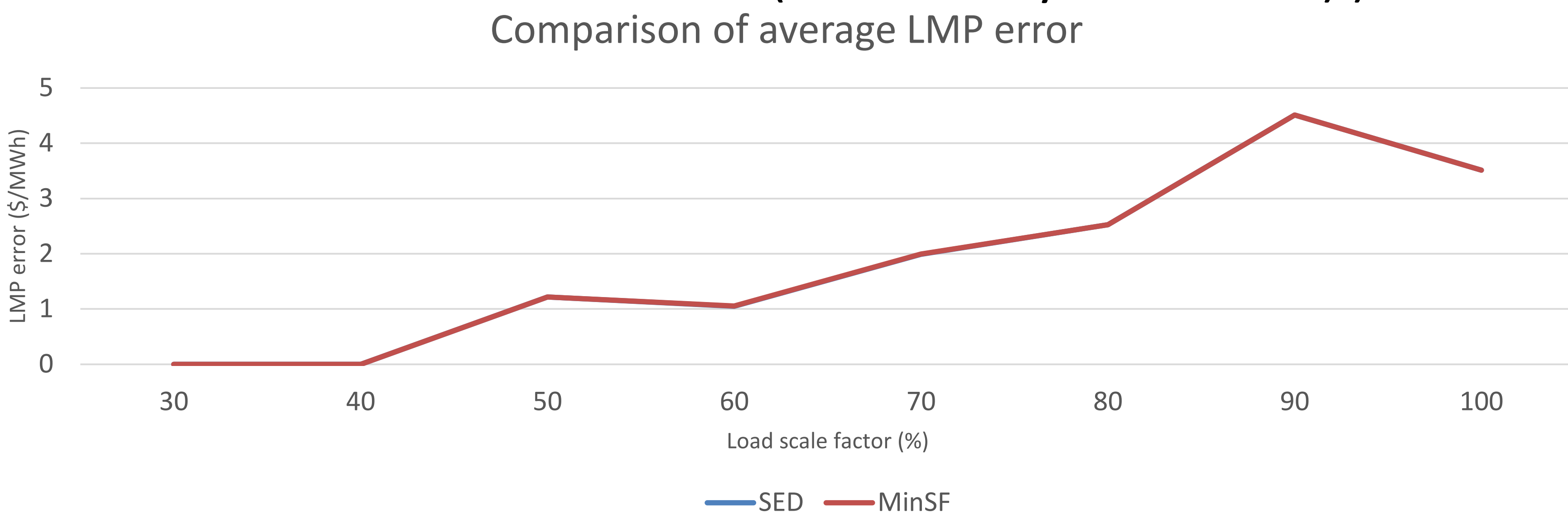


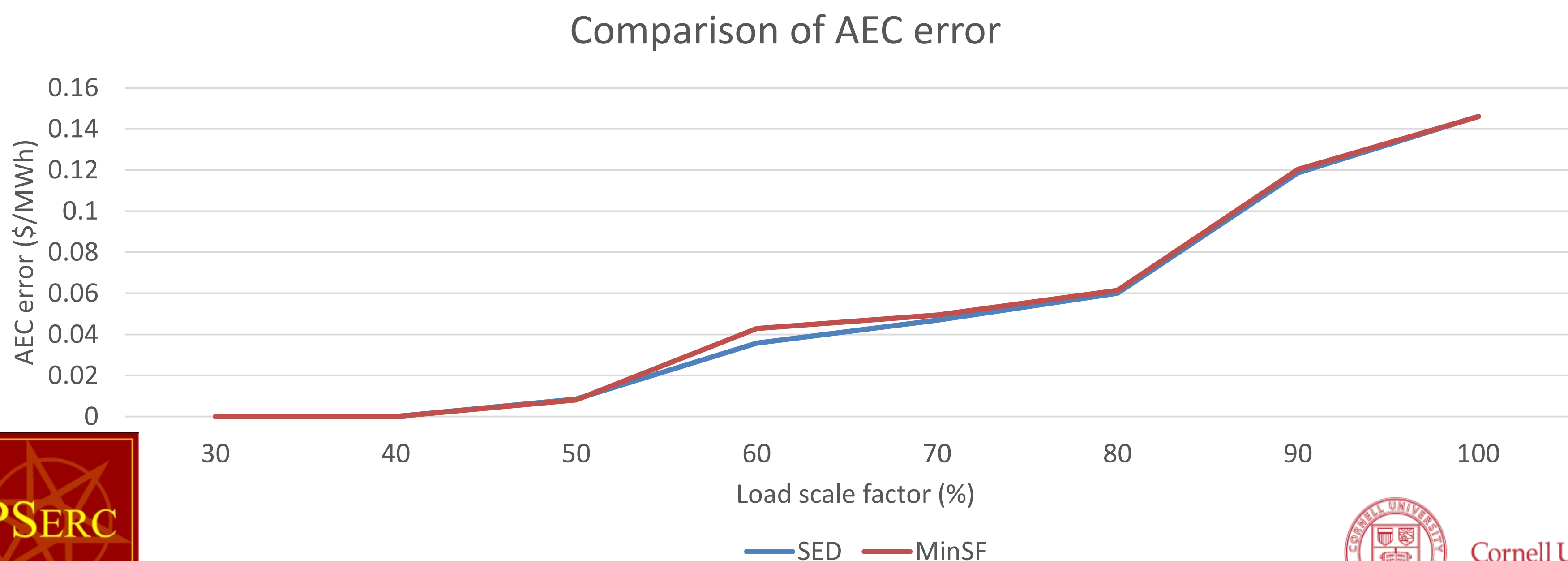
Generator Placement

- Next more aggressive reductions on ERCOT, WECC and EI systems were tested where the systems were reduced to about one tenth of their original size.

Generator Placement

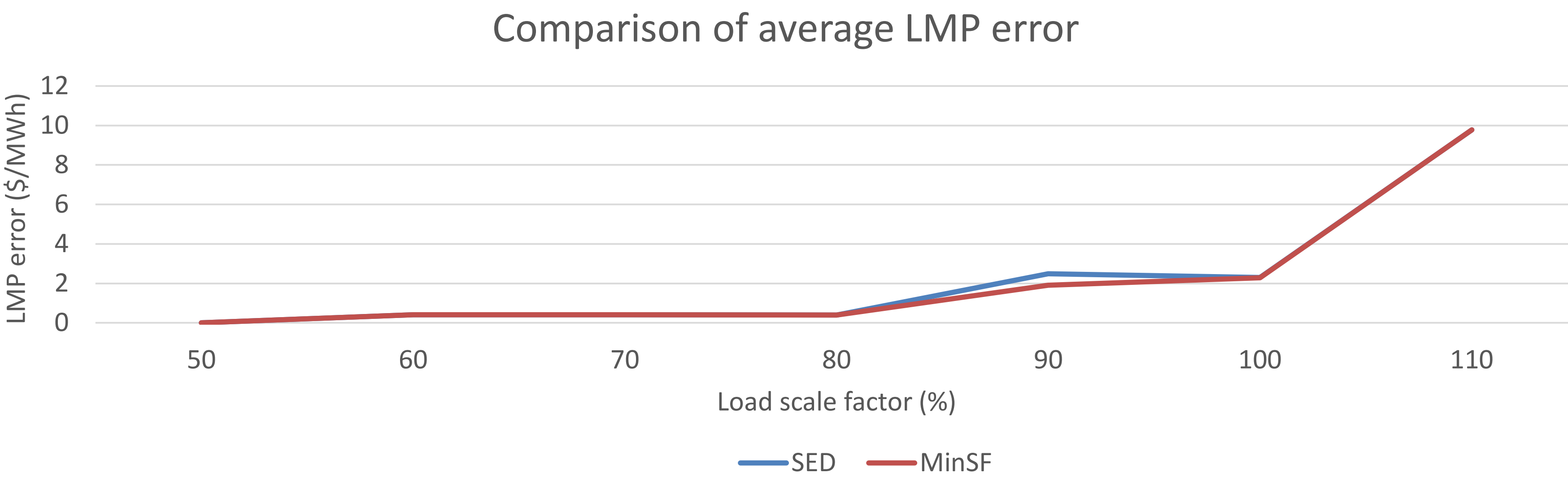
Results of WECC (2000 bus system ≈10%))

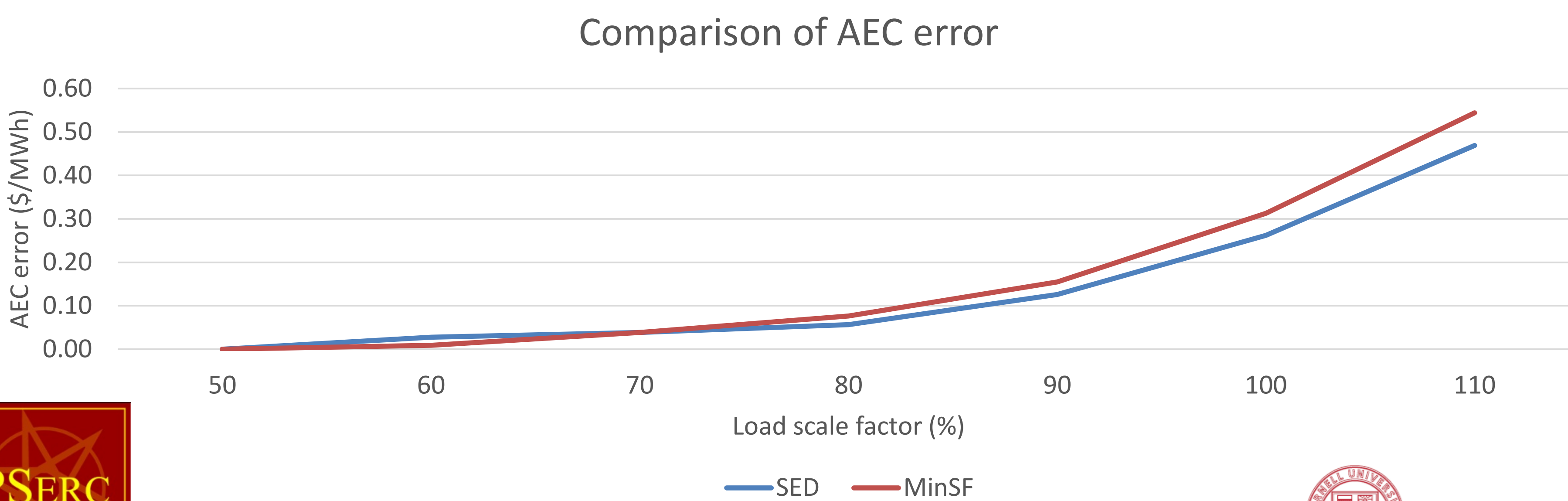




Generator Placement

Results of ERCOT (424 bus system ≈10%))





Generator Placement

- Conclusion: The two placement methods yielded similar results.
- Conclusion: Systems cannot be reduced indefinitely without consequences to accuracy.

Network Reduction Toolbox

- Last year:
- Sparsity technique was not sufficiently used in the beta release and resulted in two drawbacks.
  - High memory demand.
  - Relatively long execution time.
- Major updates:
  - Rewrote the algorithm of the partial LU factorization so that the reduced model can be constructed during the factorization process.
  - Improved symbolic processing of sparsity pattern of the reduced model.

Network Reduction Toolbox

- Efficiency before and after the update

Case	# of buses		Calculation Time	
	Unreduced	Reduced	Before Update	After Update
ERCOT	6000	424	3.5 min	25 sec
WECC	17000	2000	3 min	20 sec
WECC	19000	300	4.2 hour	2.4 min
EI	62000	5222	Out of Memory	1.3 hour

- Computation Environment:
  - Run on Matlab 2014a.
  - CPU Intel Core I7 3770, 3.4 GHz.
  - 16 GB DDR 3 memory.

Network Reduction Toolbox

- Network Reduction Toolbox Distribution
  - The toolbox is distributed along with MATPOWER 5.1.
  - The toolbox is also available on the E4ST website. http://e4st.com/
- The toolbox is currently used by the Ben Hobbs' group to do a study on transmission expansion in WECC system.

Transmission Expansion

- Assisting Cornell group in identifying transmission expansion projects for comparison.
- Proposed three candidate transmission lines
  - #1 Quebec – New York (Champlain-Hudson Power Express)—Bill Schulze
  - #2 Southern California – Arizona
  - #3 Manitoba – Minnesota

Transmission Expansion

- Candidate #1: Champlain Hudson Power Express
- This project is a 1000 MW HVDC line.
- Currently it is being studied by the E4ST research group.
- Connecting Hertel substation in La Prairie with New York City.

ARIZONA STATE UNIVERSITY

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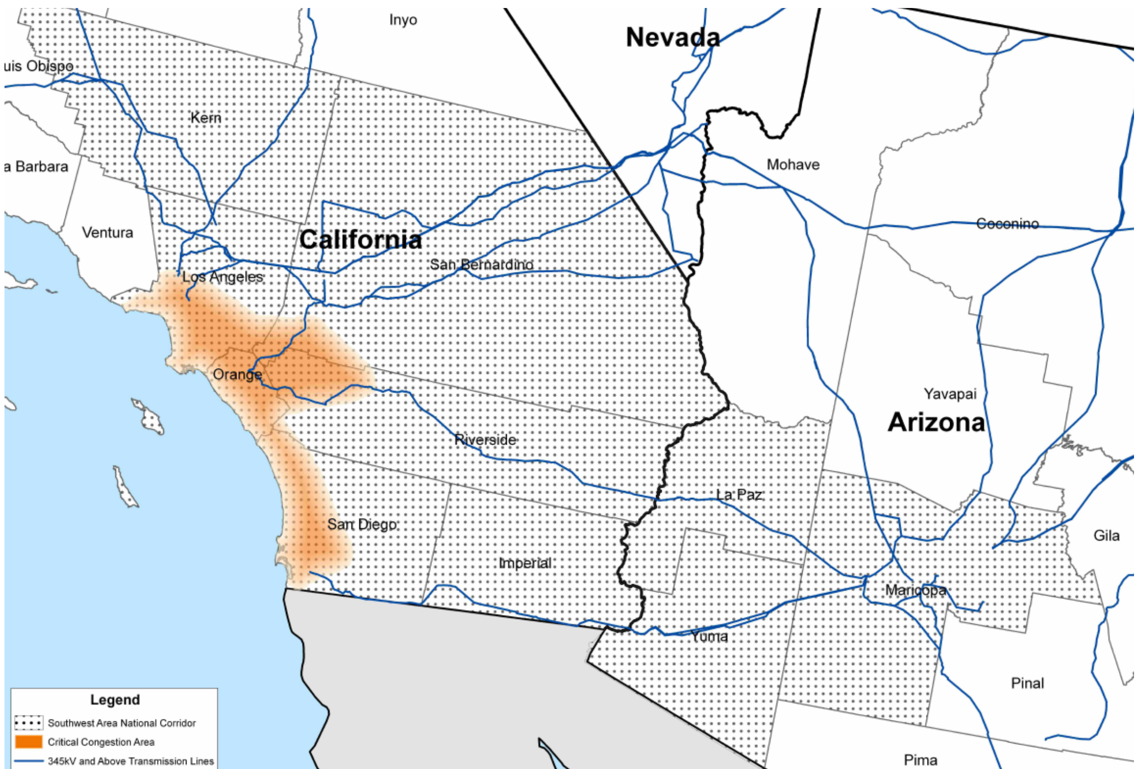
Transmission Expansion

- Candidate #2: Southern California - Arizona
- Facts:
  – Within national congestion corridor defined by the 2006 and 2009 National Electric Transmission Congestion Study.



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Transmission Expansion

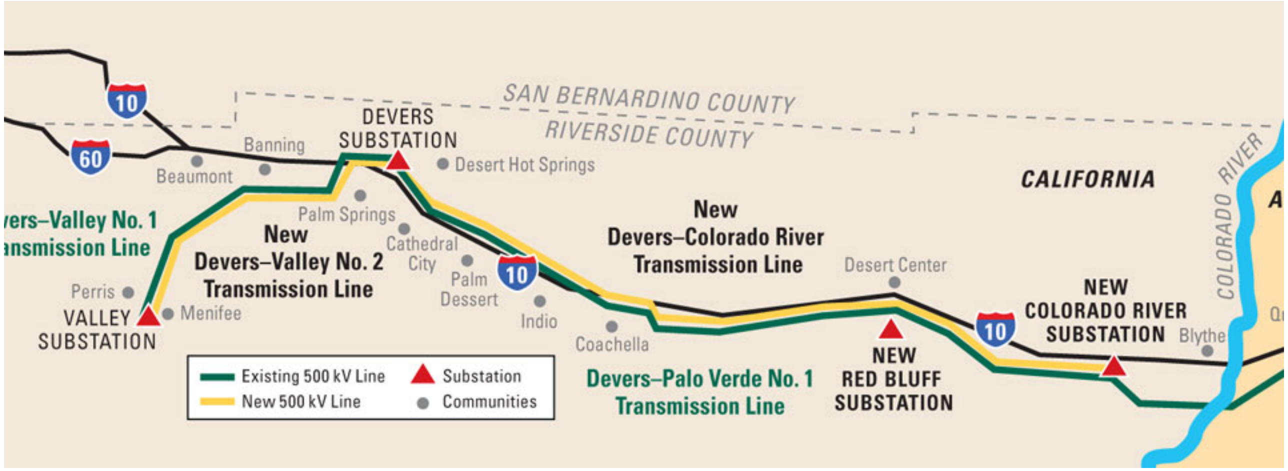
- Candidate #2: Southern California – Arizona
- Southern California Edison (SCE) in Apr. 2005 proposed 500 kV ac transmission line project (DPV2) the Devers-Palo Verde No. 2.
- The project was approved on California side by California Public Utilities Commission (CPUC).

Transmission Expansion

- Candidate #2: Southern California – Arizona
- On Arizona side, the project was denied by Arizona Corporation Commission (ACC) in June 2007.
- The major concern is that the ACC believed that the proposed transmission line will lower the rate on California side however raise the rate in Arizona.

Transmission Expansion

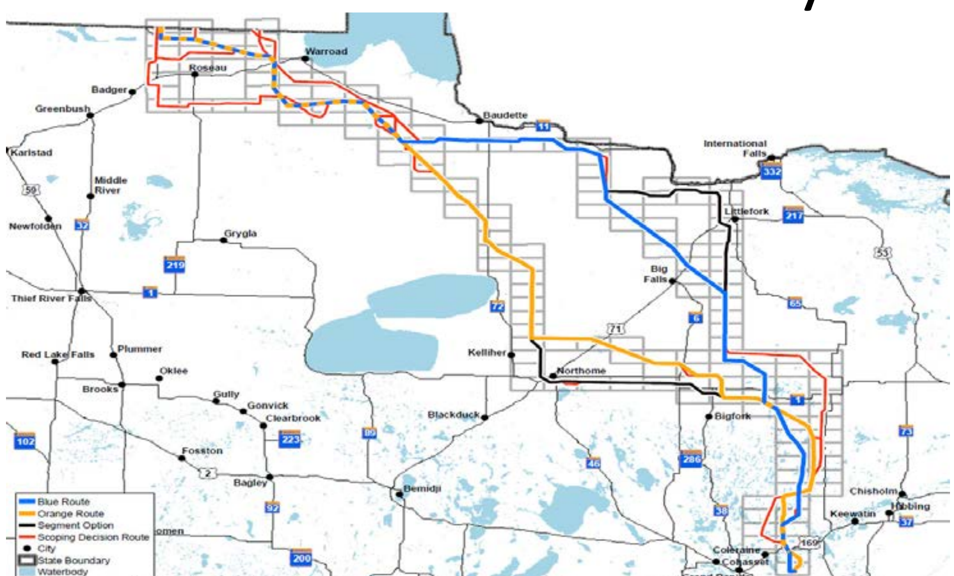
- Candidate #2: Southern California – Arizona
- Current status:
  - The construction of California portion is completed.



California portion of DPV2 project

Transmission Expansion

- Candidate #3: Manitoba – Minnesota
- Facts:
  - The Great Northern Transmission Line (between Manitoba Hydro and Minnesota Power) a 500 kV ac transmission line between province of Manitoba in Canada and Blackberry Sub. in Itasca County.



Transmission Expansion

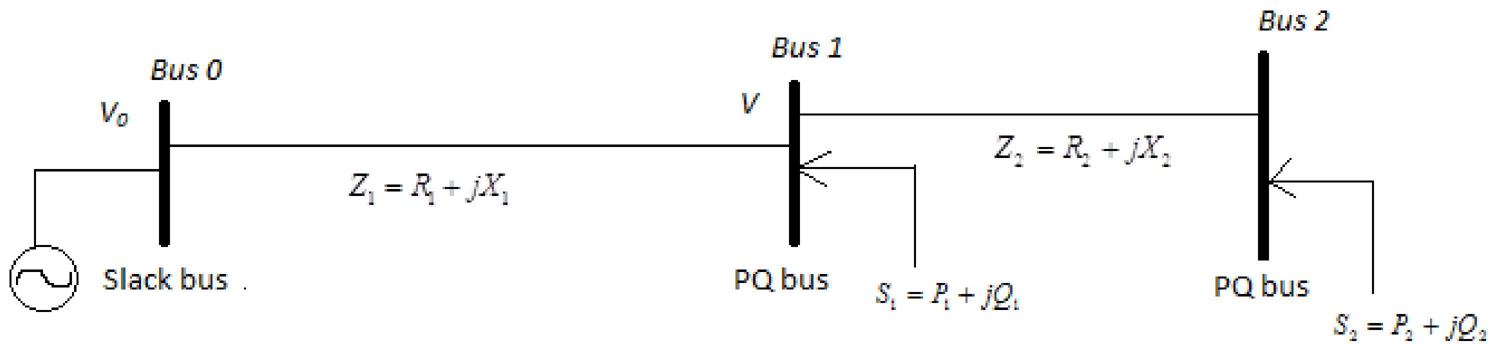
- Candidate #3: Manitoba – Minnesota
- Status
  - The project was proposed in 2012 and is currently under federal and state review.
  - On June 30, 2015 the Minnesota Public Utilities Commission (PUC) issued a written order for a Certificate of Need for the Great Northern Transmission Line.
  - More capability to deliver clean power.
    - Hydro power to be delivered from Manitoba.
    - Wind power to be delivered from Minnesota.
  - Improve system reliability.

Inverse Function Network Reduction

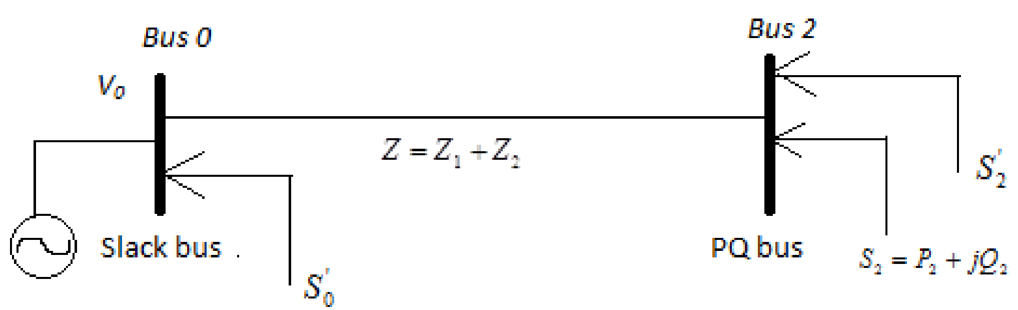
- Traditional (e.g., Ward-type and REI) reduction methods:
  - Linearize nonlinear (PQ) loads at external buses:
    - Impedances
    - Current Injections
  - Distribute linear loads via reduction rules.
  - Convert linear to equivalent nonlinear (PQ) loads at base case loading.
- Do not handle nonlinear (PQ) loads accurately because of complexity of nonlinear reduction.
- Examine whether retaining a nonlinear model in the reduction process was important for ac bus voltage preservation.

Inverse Function Network Reduction

- Consider a three-bus network as shown below.



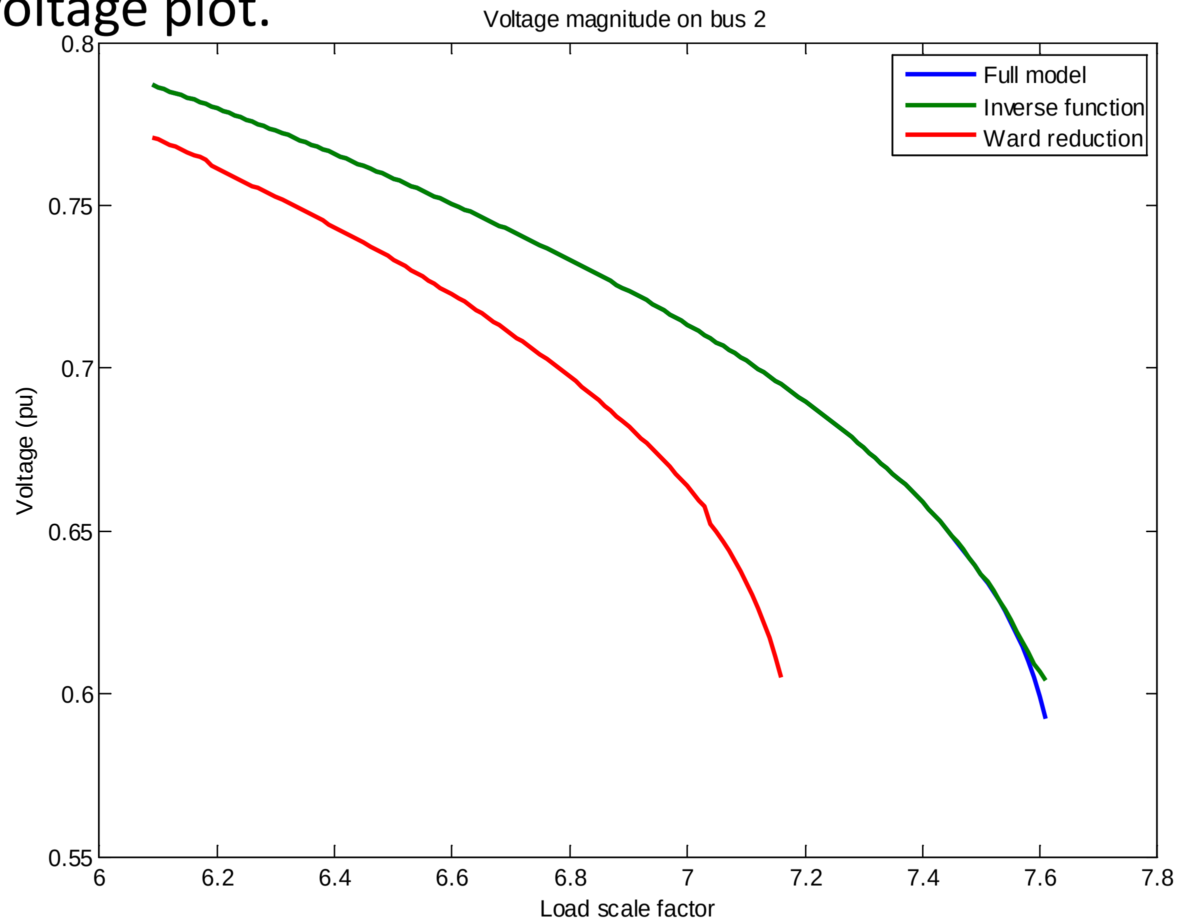
- Ward reduction: Convert PQ load at bus 1 to current injections
- Eliminate bus 1 using Ward reduction method—split I_1 between buses 0 and 2.
- Convert current injections to equivalent S=P+jQ loads at buses 0 and 2.



- Accuracy Test: Scale loads uniformly to the voltage collapse point.

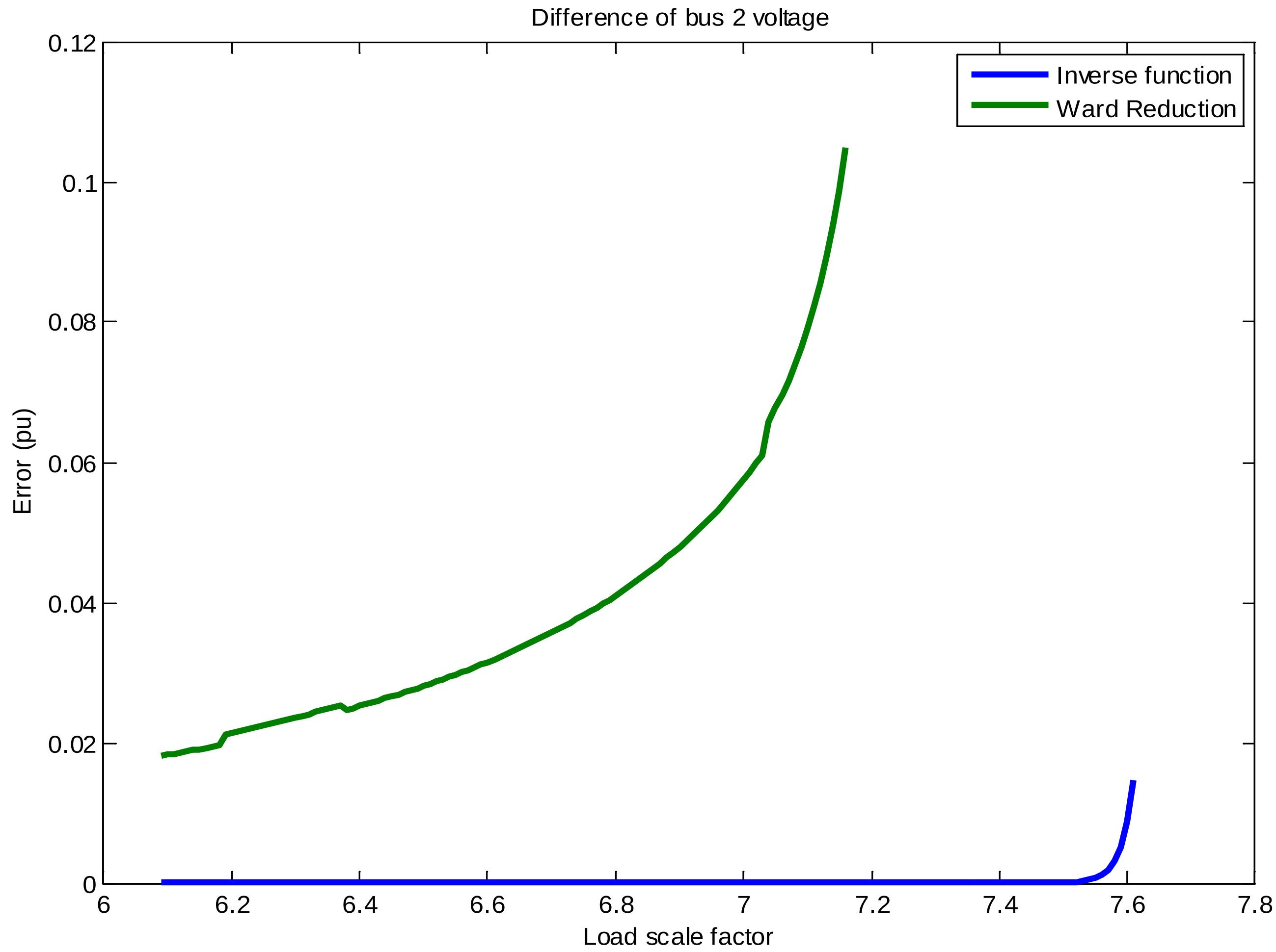
Inverse Function Network Reduction

- Static voltage collapse point:
  - Unreduced Network: VC=7.63×Base_Load
  - Inverse Function Approach: VC=7.61
  - Ward Reduction: VC=7.17
- Bus 2 voltage plot.



Inverse Function Network Reduction

- Bus 2 voltage error plot.



Inverse Function Network Reduction

- Linear case.
  - *Ax=b* b(A,x) (A=admittance matrix, x=voltage, b=current injection.)
  - Inverse function: x(A, b) (Voltage as a function of current injections.)
  - Network Reduction: A(x,b) (Admittance matrix as a function of loads.)

$$Ax = b$$

$$\Rightarrow (I + D)x = b$$

$$\Rightarrow x = -Dx + b$$

- Holomorphically embed the recursion relation with parameter α.

$$x(\alpha) = -\alpha D x(\alpha) + b$$

- Represent $x(\alpha)$ as a power series in α.

$$x(\alpha) = x[0] + x[1]\alpha + x[2]\alpha^2 + \ldots + x[N_T]\alpha^{N_T}$$

- Equate corresponding powers of α on both sides of the equation.

$$x[0] = b$$

$$x[1] = -Dx[0]$$

$$\vdots$$

$$x[N_T] = -Dx[N_T - 1]$$

Inverse Function Network Reduction

- Equate corresponding powers of α on both sides of the equation.

$$x[0] = b$$
$$x[1] = -Dx[0]$$
$$\vdots$$
$$x[N_T] = -Dx[N_T - 1]$$

$$x(\alpha) = x[0] + x[1]\alpha + x[2]\alpha^2 + \ldots + x[N_T]\alpha^{N_T}$$

- Use Padé approximate to represent $x(\alpha)$ as rational approximant.

$$x(\alpha) = x[0] + x[1]\alpha + x[2]\alpha^2 + \ldots x[L+M]\alpha^{L+M} + O(\alpha^{L+M+1})$$
$$= \frac{a_0 + a_1\alpha + a_2\alpha^2 + \ldots + a_L\alpha^L}{b_0 + b_1\alpha + b_2\alpha^2 + \ldots + b_M\alpha^M} = \frac{a(\alpha)}{b(\alpha)}$$
$$= x(\alpha D, b)$$

- Last step is to get: $D(x, b)$
- Trickier for a nonlinear problem.
- Holomorphic Series Method (HSM)

Voltage-Preserving Network Equivalents using the HSM

- In the past, we have developed network equivalents that preserve branch flows for dc network power flow formulations.
- Preserve the bus voltage magnitude and angle in ac network reductions using this holomorphic series method (HSM).
- This is of particular interest for studies involving voltage stability.

Holomorphic Series Method (HSM)

- Use HSM to obtain the voltages as a function of the current and/or complex power injections, i.e., find the inverse function.
- The power balance eq. (PBE) for a *PQ* bus can be written as:
  $$\sum_{k=1}^{N} Y_{ik} V_k = \frac{S_i^*}{V_i^*}$$
- To use the HSM, first the above equation can be holomorphically embedded as follows: $$\sum_{k=1}^{N} Y_{ik} V_k(\alpha) = \frac{\alpha S_i^*}{V_i^*(\alpha^*)}$$
- With this embedding, α scales complex load, S.
- Next $V(\alpha)$ is represented as its Maclaurin series expressed as: $V(\alpha) = V[0] + V[1]\alpha + V[2]\alpha^2 + \ldots + V[N_T]\alpha^{N_T}$ with N_T number of terms in the series.

Holomorphic Series Method (HSM)

- The inverse voltage function on the RHS of the holomorphically embedded equation can be expressed as an inverse series *W*(*α*) where $$W_i(\alpha) = \frac{1}{V_i(\alpha)}$$
- Thus the PBE is represented as:

$$\sum_{k=1}^{N} Y_{ik}(V_k[0] + V_k[1]\alpha + V_k[2]\alpha^2 + \ldots + V_k[N_T]\alpha^{N_T}) =$$

$$\alpha S_i^*(W_i^*[0] + W_i^*[1]\alpha + W_i^*[2]\alpha^2 + \ldots + W_i^*[N_T]\alpha^{N_T})$$

- The solution at *α*=0 (germ) and is obtained by equating the constant terms: $$\sum_{k=1}^{N} Y_{ik} V_k[0] = 0$$
- Subsequent series terms obtained through a recurrence relation obtained by equating like powers of *α* on both sides.

$$\sum_{k=1}^{N} Y_{ik} V_k[n] = S_i^* W_i^*[n-1]$$

Holomorphic Series Method (HSM)

- Similarly the equations for PV buses can be embedded as follows:

$$\sum_{k=1}^{N} Y_{ik} V_k(\alpha) = \frac{\alpha P_i - jQ_i(\alpha)}{V_i^*(\alpha^*)} \qquad V_i(\alpha) * V_i^*(\alpha^*) = \left| V_i^{sp} \right|^2$$

where P_i is the known power injected into the bus and V_i^{sp} is the specified voltage for the PV bus.

- The embedded equation for the slack bus is given by:

$$V_{slack}(\alpha) = V_i^{sp}$$

- The terms of the voltage series for the PV buses can be obtained in a similar manner as that for PQ buses:

$$\sum_{k=1}^{N} Y_{ik} V_k[n] + j\left(Q_i[n] W_i^*[0] + Q_i[0] W_i^*[n]\right)$$

$$= P_i W_i^*[n-1] - j \sum_{k=1}^{n-1} Q_i[k] W_i^*[n-k]$$

Holomorphic Series Method (HSM)

- The voltage magnitude constraint ultimately leads to:

$$V_i[0]V_i^*[n] + V_i[n]V_i^*[0]$$
$$= -(V_i[1]V_i^*[n-1] + \ldots + V_i[n-1]V_i^*[1])$$

- Combining the slack, PQ and PV bus equations, the PBE's of a power system can be solved recursively to obtain the terms of the voltage power series.
- Challenge: The voltage power series may not always converge.
- Padé approximants are used to obtain a converged solution, if it exists.

Padé approximants

- Stahl's Padé convergence theory- For an analytic function with finite singularities, the sequence of near-diagonal Padé approximant converges to the function... [1]
- Padé approximants are rational approximants to the given power series given by:

$$V(\alpha) = V[0] + V[1]\alpha + V[2]\alpha^2 + \ldots V[L+M]\alpha^{L+M} + O(\alpha^{L+M+1})$$

$$= \frac{a_0 + a_1\alpha + a_2\alpha^2 + \ldots + a_L\alpha^L}{b_0 + b_1\alpha + b_2\alpha^2 + \ldots + b_M\alpha^M} = \frac{a(\alpha)}{b(\alpha)}$$

[1] H. Stahl, "On the Convergence of Generalized Padé Approximants," Constructive Approximation, 1989, vol. 5, pp. 221–240.

Estimating Voltage Collapse Point (VCP) from Roots of the Padé Approximant

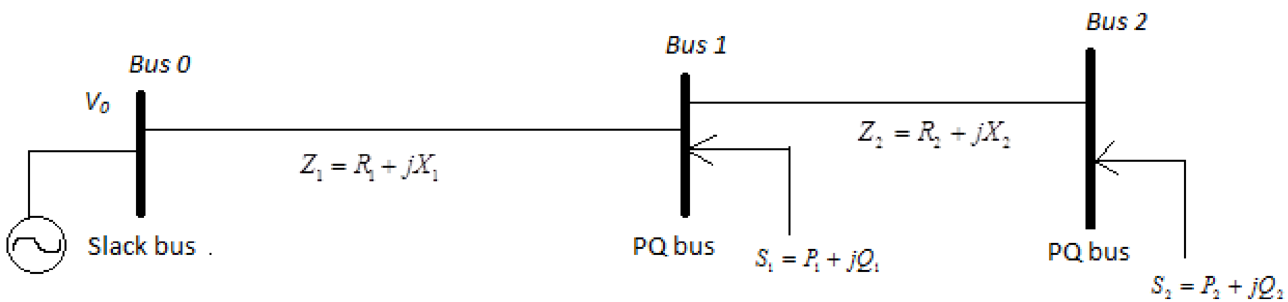
- Need to know the limits over which the Pade' approximant is valid.
- VCP estimate is the smallest real zero of the numerator or denominator polynomials of the Padé approximants of *any* bus voltage[1].[2]

1. A formulation such that the solution at different values of α represents the solution at different loading levels of the system, must be used.

[2] George A. Baker, Jr., Peter Graves-Morris, "Padé approximants," Cambridge University Press, 1996

Inverse Function Network Equivalents

- Once the voltage series for a given power flow problem are obtained, can develop reduced radial networks whose branch admittances are represented as a power series.
- Let the reduced system include the slack bus and any two buses from a large system. (Note that the topology is arbitrary.)



Inverse Function Network Equivalents

- To find branch admittances as functions of α, $Y_{ik}(\alpha)$, for the reduced network, use the voltage series of the retained buses in the PBEs.

$$\sum_{k=1}^{N} Y_{ik}(\alpha)V_k(\alpha) = \frac{\alpha S_i^*}{V_i^*(\alpha^*)}$$

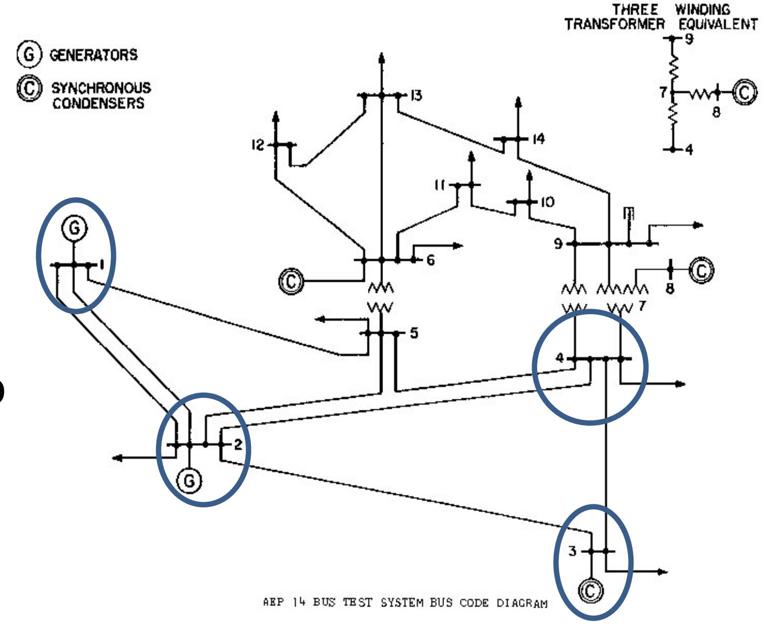
- The admittance and voltage variables in the above equation are expanded into power series as:

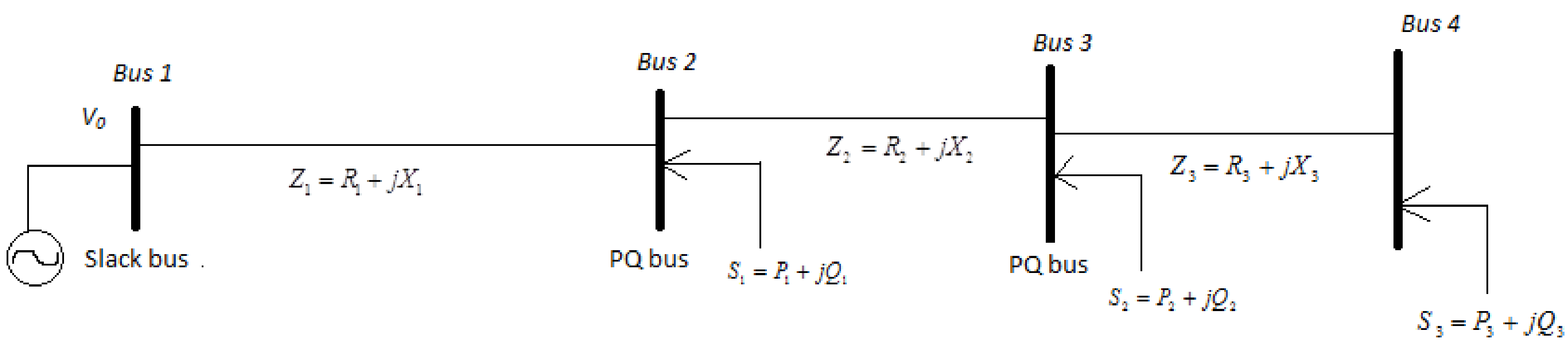
$$\sum_{k=1}^{N}((Y_{ik}[0]+Y_{ik}[1]\alpha+Y_{ik}[2]\alpha^2+\ldots+Y_{ik}[N_T]\alpha^{N_T}))(V_k[0]+V_k[1]\alpha+V_k[2]\alpha^2+\ldots+V_k[N_T]\alpha^{N_T})$$
$$=\alpha S_i^*(W_i^*[0]+W_i^*[1]\alpha+W_i^*[2]\alpha^2+\ldots+W_i^*[N_T]\alpha^{N_T})$$

- Equate the same powers of α on both sides of the equation, to find the *Y* series.
- This reduced network more faithfully preserves the voltages.

Results of the HSM generated network equivalent

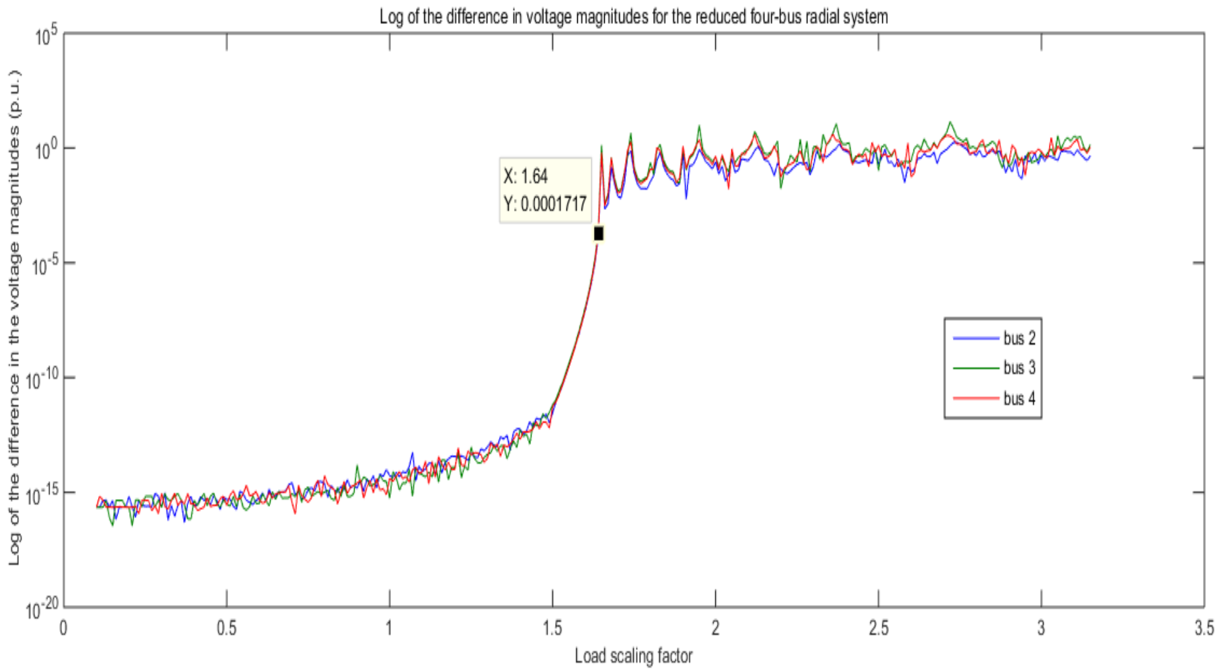
- Tested the approach for systems with PQ buses only.
- For the IEEE 14 bus system, the four PV buses (2,3,6 and 8) were converted to PQ buses and a reduced radial network was generated with the slack bus connected to bus 2, 2 to 3 and 3 to 4.





Results of the HSM generated network equivalent

- Plot: log of voltage error v. load scaling factor.



- Voltage collapse point scaling factor for the original network = 1.68

PSERC

Cornell University

Lunchtime