Energy Indicators System: Index Construction Methodology

1. Introduction

The system of energy intensity indicators is designed to track the changes in energy intensity for the total U.S. economy, broad end-use sectors, and various sub-sectors. The system employs a hierarchal structure in which time series indexes at lower levels of the hierarchy are used to generate indexes for higher levels.

A factorization method is used to develop three types of indexes that explain the change in energy use over time for each level of the hierarchy:

- *Activity* index that shows the changes in the level of activity for a sector of the economy. The units used to measure activity differ by sector (e.g., square footage of floor space, industrial production measured in dollars, passenger-miles, etc.)
- *Component-based energy intensity* index that represents the effect of changing energy intensity for sub-sectors or detailed components of the economy.
- *Structural* index that shows the effect of changing economic structure. This index is employed at higher levels of the indicators hierarchy and reflects the impact on energy use from changes in the relative importance of sectors at lower levels of the hierarchy. It primarily shows the impact of shifts in the composition of sectors or sub-sectors with different absolute energy intensities.

The component-based energy intensity index is similar in concept to the consumer price index (CPI). The CPI is based upon an aggregation of prices for many goods and services in the economy, the importance of any specific price depends upon the share of expenditures for that good or service across all consumers. The energy intensity indexes are based upon aggregations of energy intensities for more specific activities. In this application to energy intensities, the relative weights in the aggregation are based upon the shares of energy consumption for different activities.

2. Indicators Hierarchy

The factorization approach used throughout the system is aimed toward providing a decomposition of changes in total energy use into key explanatory factors. As mentioned in the introduction, this decomposition yields a set of index numbers for three distinct effects—activity, structure, and intensity—whose multiplicative product equals the index of energy use.

The system of energy intensity indicators is organized in a hierarchal manner. That is,

Table 1. Structure of Energy Intensity Indicators

Level : 0	1	2	3	4	5	Activity
Economy-wide						GDP (1996$)
	Residential					Households (HH) & Floorspace (Sq. Ft. = SF)
		Northeast				
		Midwest				
		South				
		West				
			Housing types (for all regions)			
			Single-family detached			
			Single-family attached			
			Mobile home			
			Multi-family (2-4 units)			
			Multi-family (> 4 units)			
	Commercial					Floor space (Sq. Ft = SF)
	Industrial					GDPind (1996$)
		Manufacturing				GDPman (1996$)
			21 NAICS sectors			GDPnaics, Shipments
		Nonmanufacturing (ag., mining, & constr.)				GDPnonman (1996$)
	Transportation					Weighted Index
		Passenger Transportation				Passenger-miles (P-M)
			Highway transportation			Passenger-miles (P-M)
				Personal vehicles		Passenger-miles (P-M)
					Automobiles	Passenger-miles (P-M)
					Light-duty trucks	Passenger-miles (P-M)
				Busses		Passenger-miles (P-M)
			Air transportation			Passenger-miles (P-M)
				Scheduled carriers		Passenger-miles (P-M)
				General aviation		Passenger-miles (P-M)
			Rail transportation			Passenger-miles (P-M)
				Urban rail		Passenger-miles (P-M)
					Commuter rail	Passenger-miles (P-M)
					Heavy rail	Passenger-miles (P-M)
					Light rail	Passenger-miles (P-M)
				Intercity rail		Passenger-miles (P-M)
		Freight Transportation				Ton-miles (T-M)
			Trucking			Ton-miles (T-M)
				Single-unit		Ton-miles (T-M)
				Combination		Ton-miles (T-M)
			Pipelines			Ton-miles (T-M)
				Natural gas		Ton-miles (T-M)
				Petroleum		Ton-miles (T-M)
			Air			Ton-miles (T-M)
			Water			Ton-miles (T-M)

Table 1. Structure of Energy Intensity Indicators (cont'd)

Level : 0	1	2	3	4	5	Activity
	Electric Power Sector					Delivered electricity (kWh)
		Electricity-only plants				Delivered electricity (kWh)
			Fossil Fuel			Delivered electricity (kWh)
				Coal		Delivered electricity (kWh)
				Petroleum		Delivered electricity (kWh)
				Natural Gas		Delivered electricity (kWh)
				Other Gases		Delivered electricity (kWh)
			Nuclear			Delivered electricity (kWh)
			Hydroelectric			Delivered electricity (kWh)
			Renewable			Delivered electricity (kWh)
				Wood		Delivered electricity (kWh)
				Waste		Delivered electricity (kWh)
				Geothermal		Delivered electricity (kWh)
				Solar		Delivered electricity (kWh)
				Wind		Delivered electricity (kWh)
		Combined Heat and Power (CHP) plants				Delivered electricity (kWh)
			Fossil Fuel			Delivered electricity (kWh)
				Coal		Delivered electricity (kWh)
				Petroleum		Delivered electricity (kWh)
				Natural Gas		Delivered electricity (kWh)
				Other Gases		Delivered electricity (kWh)
			Renewable			Delivered electricity (kWh)
				Wood		Delivered electricity (kWh)
				Waste		Delivered electricity (kWh)
			Other			Delivered electricity (kWh)

indexes are developed for detailed sub-sectors of the economy and build upward to create indicators for more aggregate sectors. The current version of this structure is shown in Table 1. The levels of the hierarchy are numbered, with the most aggregate, economy-wide, indicators set as Level 0. At present, the most developed structure is in the transportation sector, where the hierarchy extends to Level 5 for the most detailed modes.

As shown in the table, the economy-wide measures of energy intensity (and structural change) are based upon an aggregation of the indexes constructed for each of major end-use sectors defined by the Energy Information Administration. The system of energy intensity indicators considers standard sectors for which EIA develops annual energy consumption information: residential, commercial, industrial, and transportation. When energy is measured in "delivered" terms (i.e., excluding an imputation for electricity generation and transmission losses), a separate electricity generation sector may also be considered.

3. Measures of Energy Intensity and Intensity Indexes

As shown in last column of Table 1, the system of energy intensity indicators generally employs commonly used measures of activity to define energy intensity. In transportation, for example, commonly used measures of intensity are energy use per passenger-mile, energy use per vehicle-mile, and energy use per ton-mile. In the industrial sector, the measures are normally energy use per dollar of production or, where available, energy per physical unit of production. In the buildings sector, energy intensity is expressed in terms of per square foot of floorspace.

At the sub-sector or component level, energy intensity is unambiguously defined as the ratio of energy use per unit of activity. Thus, if E_i is the energy use for component i and A_i is the activity for component I, the component-based intensity is defined as

$$I_i = E_i / A_i \qquad (1)$$

When *two or more* components or sub-sectors for which the activity is measured on a common basis are aggregated, two types of energy intensity can be distinguished. The first is termed an *aggregate* energy intensity, defined simply as the ratio of total energy divided by total activity. The aggregate intensity I is defined as

$$I = \frac{\sum E_i}{\sum A_i} \qquad (2)$$

where E_i = energy consumption in component i
A_i = activity level in component i

The second type of aggregate intensity is defined as an index number that only can be used to measure the change in intensity from a specific base period. In this manner, it is similar to the CPI, an index that represents a measure of the overall change in prices relative to a particular time period. In the energy intensity application here, this second index will often referred to as a component-based intensity index (I_{int}), as it is based upon on a direct aggregation (or function) of the individual component energy intensity indexes. The simplest type of index formulation uses constant weights (that sum to 1.0) to construct the component-based intensity index in period *T* relative to period 0:

$$I_{\text{int}} = \sum_i w_i (I_{i,T} / I_{i,0}) \qquad (3)$$

Clearly, at any level where activities can be added in common units, the aggregate energy intensity can also be expressed in physical units. Thus, for example, we can define aggregate passenger energy intensity (a Level 2 intensity as shown in Table 1) in terms of energy consumption per passenger-mile for all modes taken together. Of course, the aggregate energy intensity can also be converted to an index that can be normalized to the same year (or any other year) as the component-based index shown in Eq. (3).

For some applications, the aggregate energy intensities are useful summary indicators, as they have either a straightforward interpretation expressed in physical units or they can be converted to a time series based index. However, changes in the aggregate intensity over time are influenced not only by changes in the energy intensities of the various components but also the relative shares of activity accounted by each of the components. These structural (or compositional) shifts are especially important when the absolute levels of the energy intensities vary significantly between components. Again, using transportation as an example, energy use per vehicle-mile for passenger vehicles (automobiles and light-duty trucks) has been affected markedly by the shift to more energy-intensive light-duty trucks over the past two decades. Changes in the (the more aggregate, Level 4) energy intensity for all personal passenger vehicles depend on both the changes in the intensities in the specific vehicle types (components) as well as shifts in relative stocks of the two types of vehicles.

Thus, in general, an index developed from the aggregate energy intensity will not be equal to the component-based intensity index. Only in the case where changes in the individual component intensities are the same, with constant shares of activity for each component, will the indexes be equivalent. In the system of energy intensity indicators, the difference between the two indexes is regarded as measure of structural change, as discussed in the following section.

Relationship of Intensity and Structural Indicators

Various types of factorization methods have been employed by which structural and compositional effects can be distinguished from the overall change in the energy intensities as represented by the component-based intensity index.[1] A key objective in the system of energy intensity indicators is the development of time series indexes that satisfy a multiplicative relationship of the general form

$$\text{Index(Energy Use)} = \text{Index (Activity)} \times \text{Index (Structure)} \times \text{Index (Intensity)} \qquad (4)$$

The indexes are normalized to a particular base year, similar to the CPI or price deflator for the Gross Domestic Product.

[1] A comprehensive survey of methods and empirical studies related to this topic is found in Ang and Zhang (2000).

As discussed above, the last term--the energy intensity index--is termed a component-based energy intensity index. This is done to distinguish this index from the aggregate energy intensity in Eq. (2).

While I in Eq. (2) can be expressed in physical units, it is easily converted to a series of index numbers by dividing each year's intensity by the intensity in a base year. In the system of economic indicators, total activity is also converted to a series of index numbers by normalizing its value to be1.0 in a particular base year. Thus, the index of aggregate energy intensity is expressed as:

$$\text{Index (Aggregate Energy Intensity } [I]) = \frac{\text{Index (Energy Use)}}{\text{Index (Activity)}} \qquad (5)$$

From Equations (4) and (5), we then see that an energy decomposition will result in a general relationship between the structure index and the alternative indexes of energy intensity as follows

$$\text{Index (Aggregate Energy Intensity}[I]) = \text{Index (Structure)} \times \text{Index (Component-based Energy Intensity)} \qquad (6)$$

While the component-based energy intensity index is normally the indicator of key interest, the system of energy intensity indicators also recognizes the value of aggregate energy intensities for some types of issues. The available spreadsheets include both types of intensity indicators and how they are related via indexes of changes in economic structure.

This issue perhaps gets the most attention at the economy-wide level, where the aggregate energy intensity is typically measured as the ratio of total energy consumption to GDP (Btu/$). One of the objectives of the system of energy intensity indicators is to show how the changes in the energy-GDP ratio can be decomposed into indicators of structural change and the change in underlying energy intensity. The specific approach to how this decomposition is undertaken is discussed later in this document.

Treatment of Dissimilar Activities

An aggregate energy intensity depends upon the notion that activities of sub-sectors or components are sufficiently alike to be expressed in a common unit. Based upon Eq. (4), we see that this situation is what in fact gives rise to a "structural" index.

However, as move up through the indicators hierarchy we clearly find that activities are sufficiently dissimilar that it makes no sense to try to aggregate them directly. One example is in passenger and freight transportation where passenger-miles and ton-miles represent significantly different bundles of services. One arbitrary approach is to attribute a (literal) weight to an average person and then convert all transportation activity to ton-miles. While this solution has some slight logic in this instance, we see from Table 1 more cases in which it is meaningless to try to convert activity measures to a common unit (e.g., square footage of floor space and chemical production measured in constant-year dollars).

As discussed in the introduction, the (component-based) energy intensity index is analogous to a price index such as the CPI. The CPI (or any other broad price index) measures overall price changes for many categories of goods and services that are very dissimilar. The key notion is that the CPI only measures *changes* in the overall level of prices from a particular period of time.   By weighting the *changes* in prices by expenditure shares of the various goods and services included in the index, the CPI can be applied across a very heterogenous mix of goods and services..

As will be shown in the next section, the component-based energy intensity index is constructed in a manner similar to a price index, only that the shares of energy consumption are used to weight the changes in intensity of each activity. Since a non-zero energy share can be estimated for any type of energy-using activity, it is clear that we can use this approach to aggregate the relative changes in energy intensity across dissimilar activities.

[Some more discussion might be inserted here to highlight the root cause of "structural change." Essentially it is an artifact of trying to explain an aggregate intensity. No such counterpart in BLS or BEA]

4. The Divisia Index Approach

The Divisia index approach allows a decomposition of the percentage change in energy use into separate changes in total activity, economic structure, and energy intensity at the component level. When applied to annual data, the decomposition is performed for changes from one year to the next. The resulting changes are cumulated to a time series index that is normalized to one in a selected base year. The chain-weighted nature of the index makes the choice of base year arbitrary from the standpoint of the percentage changes over time.

Divisia indexes assume that the data on the various factors vary in more or less a continuous fashion. Liu et al. (1998) indicate more formally that the data are assumed to be available at every moment of time t, instead of only at discrete (annual, quarterly, etc.) points in time.

As Liu et al. point out, the Divisia index has many desirable properties that are useful for decomposition analysis. These properties include variable weighting over time and additive decomposition of relative growth rates. However, decomposition based upon a general Divisia index approach does not yield a unique set of results because one can develop an infinite number of indexes, each corresponding to assumptions as how the factors change between the observed discrete points in time. Below we lay out the general framework underlying the Divisia method and discuss how several methods have emerged that provide specific results.

Energy-weighted Rates of Change

Loosely following Lermit and Jollands (2001), we consider the rate of change of total energy use for a particular sector of the economy. Continuing with the notation of the previous section, total energy (E) can be expressed as the sum of the energy use for each of the components or sub-sectors within this sector. Typically, the component intensity is defined in terms of the available data as E_i/A_i.). Thus, energy use for each component i can be represented as the product of activity (A_i) and the energy intensity in component i (I_i). Formally, we have:

$$E = \sum_i E_i = \sum A_i I_i \qquad (7)$$

At this point, assume that the activities for each of the components are measured in similar units (e.g., dollars, passenger-miles, etc.). Thus, total sector activity A is the sum of the activity levels for the components.

If we express the share of the total sector's activity for component i (A_i/A) as S_i, Eq. (7) can be rewritten as:

$$E = \sum_i AS_i I_i \qquad (8)$$

The derivative of Eq. (8) with respect to time is

$$\frac{dE}{dt} = (\sum_i S_i I_i \frac{dA}{dt} + \sum_i AI_i \frac{dS_i}{dt} + \sum_i AS_i \frac{dI_i}{dt} \qquad (9)$$

If we divide both sides of Eq. (9) by E, and perform some manipulation of each of the terms on the right side, the entire expression can be recast in terms of logarithms or percentage growth rates of each of the variables:

$$\frac{1}{E}\frac{dE}{dt} = \frac{d\ln E}{dt} = \frac{1}{E}(\sum_i AS_i I_i \frac{d\ln A}{dt} + \sum_i AS_i I_i \frac{d\ln S_i}{dt} + \sum_i AS_i I_i \frac{d\ln I_i}{dt} \qquad (10)$$

Because

$$\frac{AS_i I_i}{E} = \frac{E_i}{E} = w_i \text{ = share of energy consumed by component i} \quad (11)$$

we can see that the growth rate of total energy is simply the energy-weighted average of the growth rate of each of the factors:

$$\frac{d\ln E}{dt} = \sum_i w_i \left[\frac{d\ln A}{dt} + \frac{d\ln S_i}{dt} + \frac{d\ln I_i}{dt} \right] \quad (12)$$

Application to Discrete Time Periods

While Eq. (12) holds instantaneously, it must be integrated over a discrete time period to yield a useable result. In general terms, this integration over the interval 0 to T generates:

$$\ln(E_T / E_0) = \sum_i \int_0^T w_i \frac{d\ln A}{dt} dt \qquad \text{[Activity effect } (D_{act})\text{]} \quad (13)$$

$$+ \sum_i \int_0^T w_i \frac{d\ln S_i}{dt} dt \qquad \text{[Structural effect } (D_{str})\text{]}$$

$$+ \sum_i \int_0^T w_i \frac{d\ln I_i}{dt} dt \qquad \text{[Intensity effect } (D_{int})\text{]}$$

The logarithmic or percentage change in total energy consumption between any two points in time (D_{eng}) can therefore be decomposed into three effects.

$$D_{eng} \sim D_{act} + D_{str} + D_{int} \quad (14)$$

An approximate solution to these integrals can be obtained by selecting an appropriate function of the end points at time 0 and T. This results in the following general expressions for each of the effects:

$$D_{act} = \left(\sum_i w_i^* \right) \ln(A_T / A_0 \quad (15a)$$

$$D_{str} = \sum_i w_i^* \ln(S_{i,T} / S_{i,0}) \quad (15b)$$

$$D_{\text{int}} = \sum_i w_i^* \ln(I_{i,T} / I_{i,0}) \quad (15c)$$

Depending upon the nature of the solution method, the three effects may not exactly sum to the total change, yielding a small residual term in Equation (14). A solution yielding no residual term is said to provide a *perfect decomposition* of the total change.

The weights w_i^* in Eqs. 15a-15c are derived by an averaging of the initial and terminal shares of energy used in each of the components. How this averaging is performed reflects an assumption about the unobserved path of the variables *A*, *S*, and *I* between the initial and end periods. The most straightforward assumption is to assume that the path is linear between the end points; in this case the weights are defined

$$w_i^* = (w_{i,0} + w_{i,T}) / 2 \qquad (16)$$

The choice of these weights results is what has been termed an Arithmetic mean Divisia approach. While easy to apply, this is an imperfect decomposition method and normally results in a small residual in Eq. (14); thus the sum of the three effects may not precisely equal the total change in energy use.

In 1997, Ang and Choi proposed a refined Divisia method that results in no residual, and thus yields a perfect decomposition of the effects. Their solution was to base the weights on what is termed a logarithmic mean function of the shares. The logarithmic mean of any two variables is defined as

$$L(\mathrm{x,y}) = (\mathrm{y} - \mathrm{x})/\ln(\mathrm{y/x}) \qquad (17)$$

As applied to the energy consumption shares, the logarithmic mean function is defined

$$L(w_{i,0}, w_{i,T}) = (w_{i,T} - w_{i,0}) / \ln(w_{i,T} / w_{i0}) \qquad (18)$$

The final weights w_i^* are based upon a normalization that ensures that they exactly sum to one:

$$w_i^* = L(w_{i,0},w_{i,T}) / \sum_i L(w_{i,0}, w_{i,T}) \qquad (19)$$

Lermit and Jollands provide a proof that this formulation of the weights yields no residual term, regardless of how the specific values of the variables vary over time.

The use of the logarithmic mean Divisia method implies that all of the variables are growing at constant growth rates between the initial and terminal periods. Thus, the method assumes that unobserved values between the two periods lie on a path defined by an exponential growth curve.

Construction of Time Series Indexes

While Divisia index decomposition can be applied over any time period,, it is applied to annual observations in this system of energy intensity indicators. The logarithmic (~ percentage) change in energy use between each pair of successive years is decomposed using the log mean Divisia method, yielding the terms shown in Equation (12). By implicitly setting the first of the two years equal to 1.0, an index number for the second year is obtained by taking the exponential of each of the terms as follows:

$$\text{Exp}\,(D_{eng}) = \text{Exp}\,(D_{act} + D_{str} + D_{int}) \quad (20)$$

$$= I_{act}) \quad \text{(Activity Index, year-over-year)}$$
$$\times\ I_{str}) \quad \text{(Structure Index, year-over-year)}$$
$$\times\ I_{int} \quad \text{(Intensity Index, year-over-year)}$$

where $I_{act} = \text{Exp}(D_{tot})$, $I_{str} = \text{Exp}(D_{str})$ and $I_{int} = \text{Exp}(D_{int})$.

For each effect, the indexes are then chained to form a time series for the available data period.

These steps are illustrated with an example in Table 2. The year-to-year logarithmic changes for energy intensity are shown in column (2) and converted to index "relatives" in column (3). A chained intensity index is created in column (4), by setting the index equal to one in year 0 and applying the year-to-year relative changes to successive pairs of years. The multiplicative nature of the chaining process implies that conversion to an alternative base year will not affect the relative changes between any selected pair of years. For example, the last column (5) in the table shows a renormalization of the base period to year 2.

Table 2. Example Chain Index

Year	Year-to-year change D_{int}	I_{int}	Index Base Year 0	Index Base Year 2
0			1.000	1.026
1	-0.030	0.970	0.970	0.996
2	0.004	1.004	0.974	1.000
3	-0.015	0.985	0.960	0.985

5. Index Aggregation across the Indicators Hierarchy

The Divisia approach provides a means by which we can aggregate the indexes from one level of the hierarchy to the next. Essentially we calculate the logarithmic changes for all of the indexes from a specific level of the hierarchy and then perform the aggregation with the weighting scheme as shown in Eqs. 15a – 15c).

Formally, if we express an index of energy use at level *m* of indicators hierarchy as I^m_{eng}, we can define the results of the index decomposition by

$$I^{m}_{eng} = I^{m}_{act} I^{m}_{str} I^{m}_{\text{int}} \quad (21)$$

Any year's actual energy use associated with this decomposition is obtained by multiplying the indexes by the energy use in the base year:

$$E^{m} = E^{m}_{0} I^{m}_{act} I^{m}_{str} I^{m}_{\text{int}} \quad (22)$$

Now, we wish to aggregate all of the indexes at one level of the hierarchy (*m*+1*)* to generate the indexes at the next highest level (*m*). If we represent the indexes of the various sub-sectors to be aggregated with superscript k, then we have

$$E^{m} = \sum_{k} E^{m+1,k}_{0} I^{m+1,k}_{act} I^{m+1,k}_{str} I^{m+1,k}_{\text{int}} \quad (23)$$

An example of such an aggregation is the energy use for total passenger transportation. From Table 1, we see that the energy use and indexes for total passenger transportation (Level 2 in indicators hierarchy) are constructed from an aggregation of the same elements for the highway transportation, airlines, and railroads (Level 3 in the hierarchy).

If the activities across the various groups to be aggregated can be measured in common units (e.g., passenger-miles), then we can define an aggregate activity for Level m (A^{m}), and a share of total activity represented by each sub-sector k at level m+1 ($S^{m+1,k} = A^{m+1,k}/A^{m}$), we can rewrite Eq. (23) as

$$E^{m} = \sum_{k} (E^{m+1,k}_{0} / A^{m+1,k}_{0}) A^{m} S^{m+1,k} I^{m+1,k}_{str} I^{m+1,k}_{\text{int}} \quad (24)$$

Since the structure of Eq. (24) is similar to that of Eq. (8), the same Divisia decomposition method can be applied to yield a solution for the logarithmic changes in four separate indexes:

$$D^{m}_{act} = \left(\sum_{k} w^{*}_{k} \right) \ln(A^{m}_{T} / A^{m}_{0}) \quad (25a)$$

$$D^{m}_{str'} = \sum_{k} w^{*}_{k} \ln(S^{m+1,k}_{T} / S^{m+1,k}_{0}) \quad (25b)$$

$$D^{m}_{str''} = \sum_{k} w^{*}_{k} \ln(I^{m+1,k}_{str,T} / I^{m+1,k}_{str,0}) \quad (25c)$$

$$D^{m}_{\text{int}} = \sum_{k} w^{*}_{k} \ln(I^{m+1,k}_{\text{int},T} / I^{m+1,k}_{\text{int},0}) \quad (25d)$$

Again, the logarithmic mean version of the Divisia method indicates that the weights *w* in Equations (25a – 25d) are all defined as logarithmic means of the energy shares of the various sub-sectors between period 0 and period T.

Equations (25a) and (25c) lead to separate structural indexes, $I^{m}_{str'}$ and $I^{m}_{str''}$, respectively. The first of these indexes can be considered an effect due to the changing composition of activity between or among the sub-sectors. A shorthand verbal designation might be the "between sub-sectors effect". The second index relates the (cumulative) structural effects *within* the sub-sectors; that is the overall effect of the changing composition of components within each of the sub-sectors. Here a shorthand label might be "within sub-sectors effect." The two indexes can be multiplied together to yield the total structural effect at this level of the hierarchy. Thus, the total structural index can be calculated as

$$I^{m}_{str} = I^{m}_{str'}\, I^{m}_{str''} \qquad (26)$$

As may be inferred from Equation (25c), the "within sub-sectors effect" is a weighted average of the structural effects from the next lower level of the indicators hierarchy. That suggests that it is possible to represent the structural effect at any level as the product of structural indexes for *all* lower levels. Such a representation would permit one, for example, to estimate the effect on the total passenger transportation intensity index resulting solely from a shift from automobiles to light-duty trucks. While this information can be teased out of the indicators spreadsheets, it is not shown in the standard tables. Carrying all of the structural effects upward to higher levels of the hierarchy both complicates the presentation of results and yields different types of results tables for the various end-use sectors. Accordingly, a consistent two-level disaggregation of structural effects, as shown in Eq. (26), was the approach chosen for the current system.[2]

Consistency in Aggregation

The indicators hierarchy is useful to many types of analysis that focus on different levels of the economy and how energy intensity has changed over time. If we are only interested in indicators of energy intensity at the top levels of the hierarchy, then it is unnecessary to construct such measures for aggregates at lower levels of the hierarchy. The Divisia index approach as well as other formulations for constructing price indexes needs only data for intensities (prices) and amount of energy (dollar expenditures) for the specific goods that make up the index. Thus, aggregate intensity index at a high level of the hierarchy (i.e., economy-wide [Level 0] or major end-use sector [Level 1]) can be constructed from the component intensities at the lowest level in each branch of hierarchy.

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[2] An exception was made in the residential sector, for which we chose to show separate structural indexes for the effect of 1) changing mix of housing types, 2) regional shifts, 3) housing unit size, and 4) weather.

A natural question is whether the aggregate index constructed in this manner is equal to the index developed by aggregation of the hierarchical indexes described above. Equality between these approaches yields a property known as "consistency in aggregation."

In a recent paper, Ang and Liu (2001) point out that that logarithmic mean Divisia method provides perfect decomposition of total energy use but is not consistent in aggregation. They then propose a modification of the Divisia method that generates this consistency. In essence, the modification requires the use of the logarithmic mean of the absolute energy use, rather than the shares of energy use in the individual components of the index.

This modification was tested in the summer of 2002 with data related to the transportation sector. The modification resulted in differences in the aggregate transportation index for 2000 (1985 = 1.0) of less than 0.001% relative to the logarithmic mean Divisia method described in Section 3. On a conceptual level, however, in cases where physical units can be used, the modified Divisia approach leads to aggregate activity measures that are not simply the sum of the activities for the components. For the sake of transparency within the indicators system, this property was deemed to be very important. For this reason, as well as the fact that the two measures produced nearly identical results, the system of energy intensity indicators employs original formulation of the logarithmic mean Divisia method.

6. Decomposition of the Energy-GDP Ratio

The ratio of total energy to total GDP is often cited as the broadest measure of energy intensity in the economy. However, the energy-GDP ratio includes a myriad of structural factors that blur its ability to credibly reflect overall changes of energy intensity across all sectors of the economy. While the system of energy intensity indicators is designed to independently measure changes in energy intensity throughout the economy, it can also be used to explore the factors that influence the energy-GDP ratio.

Assume that indexes of energy intensity and structural change have been computed for all Level 1 activities, that is the major end-use sectors defined by EIA. These indexes are based upon the decomposition analysis at the lower levels of the hierarchy and following the notation of Eq. (21) from above, energy use in *each* broad end-use sector can be represented at the product of three indexes:

$$I_{eng}^{1} = I_{act}^{1} I_{str}^{1} I_{\text{int}}^{1} \qquad (27)$$

where superscript 1 indicates that these indexes are at level 1 of the indicators hierarchy. Total energy use in the economy, E^0, is represented by

$$E^{0} = \sum_{k} E_{0}^{1,k} I_{act}^{1,k} I_{str}^{1,k} I_{\text{int}}^{1,k} \qquad (28)$$

Here the index k ranges from 1 to 4, corresponding to the residential, commercial, industrial, and transportation sectors. (In some applications, the aggregation also includes a fifth sector, the electric power sector).

Since $I_{act}^{1,k}$ is equal to the total sector's activity in the current year divided by the activity in the base year, we can rewrite Eq. (28) as

$$E^0 = \sum_k (E_0^{1,k} / A_0^{1,k}) A^{1,k} I_{str}^{1,k} I_{\text{int}}^{1,k} \tag{29}$$

At the economy-wide level the activity measures for the broad end-use sectors do not sum to GDP. With the exception of the industrial sector, they are not measured in (constant) dollar terms. Moreover, some activities are not really included in the GDP, namely household energy-using equipment and household transportation activity.[3]

Nevertheless, we can represent one aspect of structural change in the economy as changes in the ratio of the end-use sector activity to GDP. Formally, we can define these ratios as

$$g^k = A^{1,k} / GDP \tag{30}$$

Incorporating Eq. (30) into Eq. (29) yields

$$E^0 = \sum_k (E_0^{1,k} / A_0^{1,k})(GDP) g^k I_{str}^{1,k} I_{\text{int}}^{1,k} \tag{31}$$

The logarithmic mean Divisia method can be applied to Equation (30) straightforwardly. In this case, there is simply an additional factor, g, to aggregate into an index—the A^k/GDP. This factor is converted into a structural index because it relates to how the sectoral activity measures change relative to the total output of the economy (GDP). In this formulation, the index of GDP essentially indicates how energy use would have grown if structure and intensity remained constant. The decomposition of the expression in Eq. (31) yields a multiplicative relationship among four indexes that holds identically:

E = Index (GDP) x Index (Structure') x Index(Structure") x Index (Intensity)

This formulation distinguishes the two sources of structural effects. The first can be termed "Activity-to-GDP" since its shows the net effect of changes in the sectoral activity measures to total GDP. The second (Structure") is based upon the Divisia decompositions made at lower levels of the indicators hierarchy and shows the impact of structural changes *within* each of the end-use sectors. A total structural change index is constructed as the product of these two indexes.

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[3] This issue was addressed at some length in a January 2001 draft paper that discussed a number of conceptual and implementation issues related to the system of energy intensity indicators.

The nature of the decomposition now allows some insight into the evolution of the energy-GDP *ratio*. By use of this method, we have

Index (energy/GDP) = Index (Structure) x Index (Intensity)

It should be clear from the preceding discussion that the size of the total structural effect depends upon the differential growth rate between the sectoral activity measures and GDP as well as structural effects within each of end-use sectors. For example, a declining ratio of households per dollar of GDP reflect general productivity improvement in the economy, as GDP per worker (and per household with a relatively constant number of workers) increases over time. But also included in the measure of overall structural change are impacts caused by compositional shifts of components or sub-sectors within each major end-use sector. For example, this aspect of structural change would include a shift between industries with high energy intensity versus low energy intensity within the industrial sector.

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